

Comparison of WarpX and GUINEA-PIG for Beam-Beam Interactions in Muon Collider

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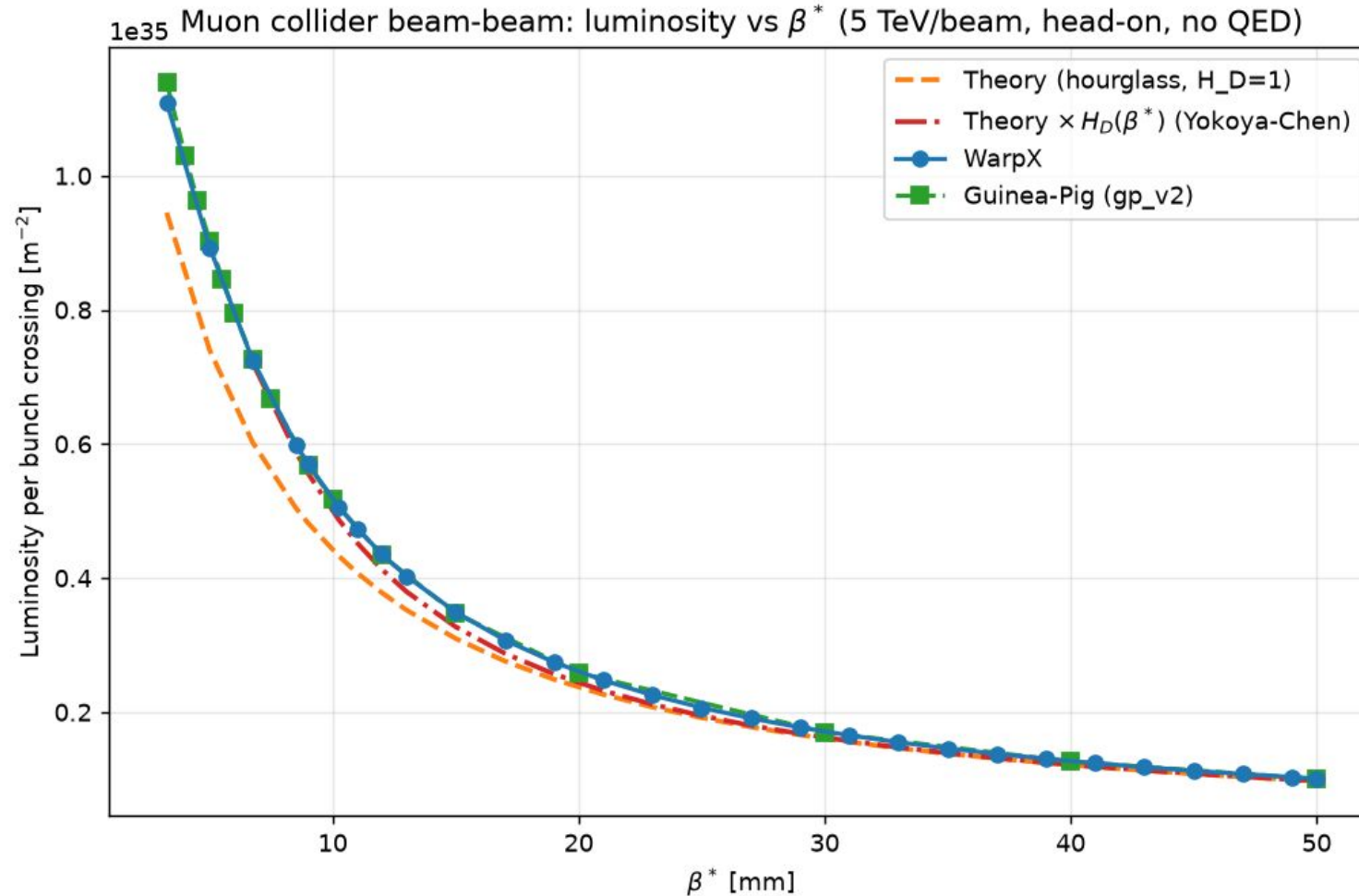
Takeaway: GUINEA-PIG and WarpX agree for muon collider beams

We were able to reach agreement between GUINEA-PIG (GP) and WarpX for beam-beam simulations in a muon collider.

- Required steps:
 - Modifying WarpX to input muons correctly
 - lots of smaller patches (see appendix) most which pertain to QED effects which end up being suppressed
 - Modifying a GP-for-muons code that was provided to me by Larry Lee, which needed debugging to
 - fix mu-/mu- to mu-/mu+
 - and correctly dump the initial muon beams

More details on modifications can be found in backup slides.

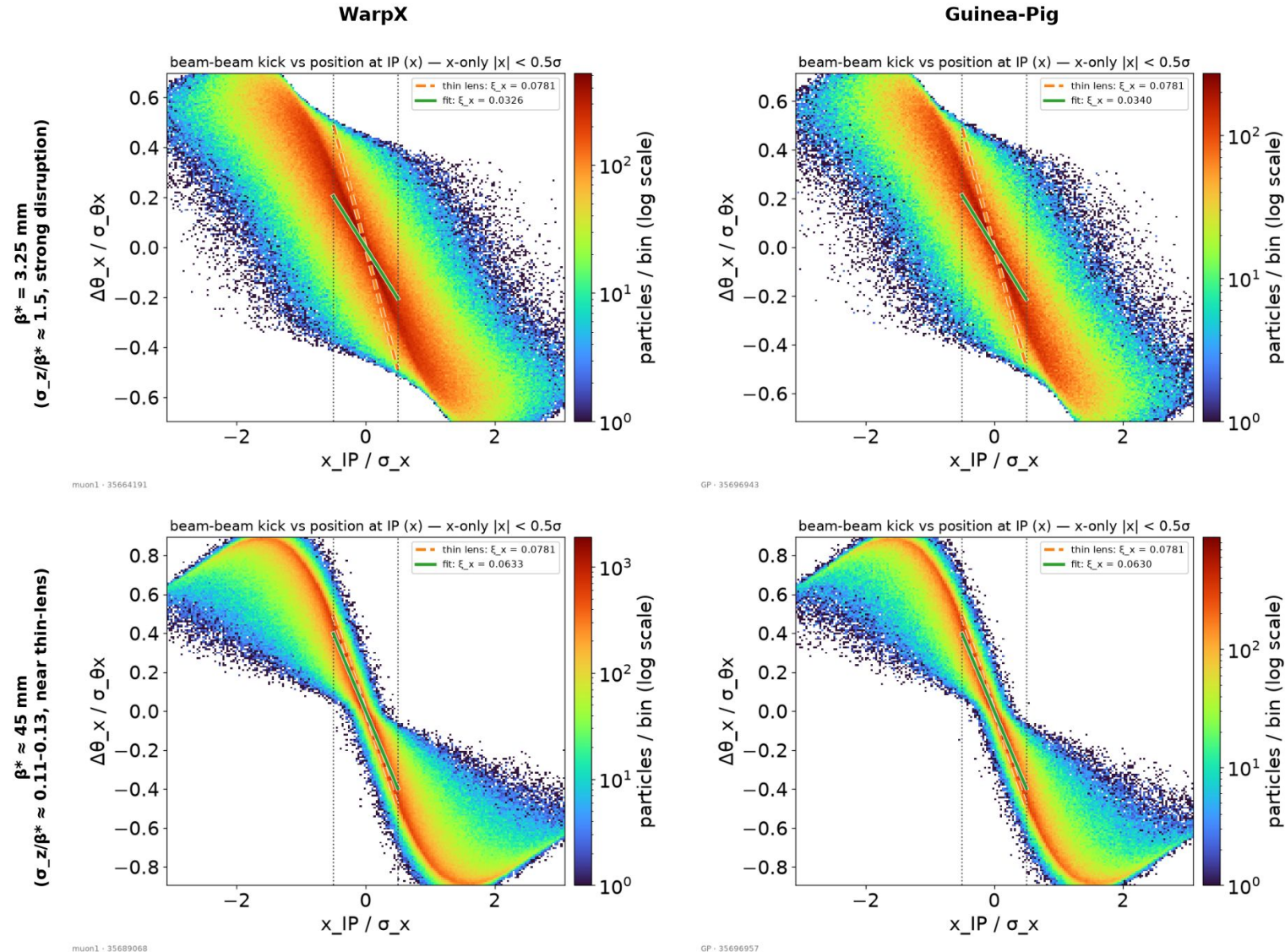
GP vs. WarpX vs. Theory: Luminosity



- GP and agree on luminosity parameter incredibly well!
- The [Yokoya-Chen 1988](#) correction to luminosity (estimated using CAIN simulations) is a more accurate estimator than geometric luminosity.
 - only valid and plotted for disruption < 0.8

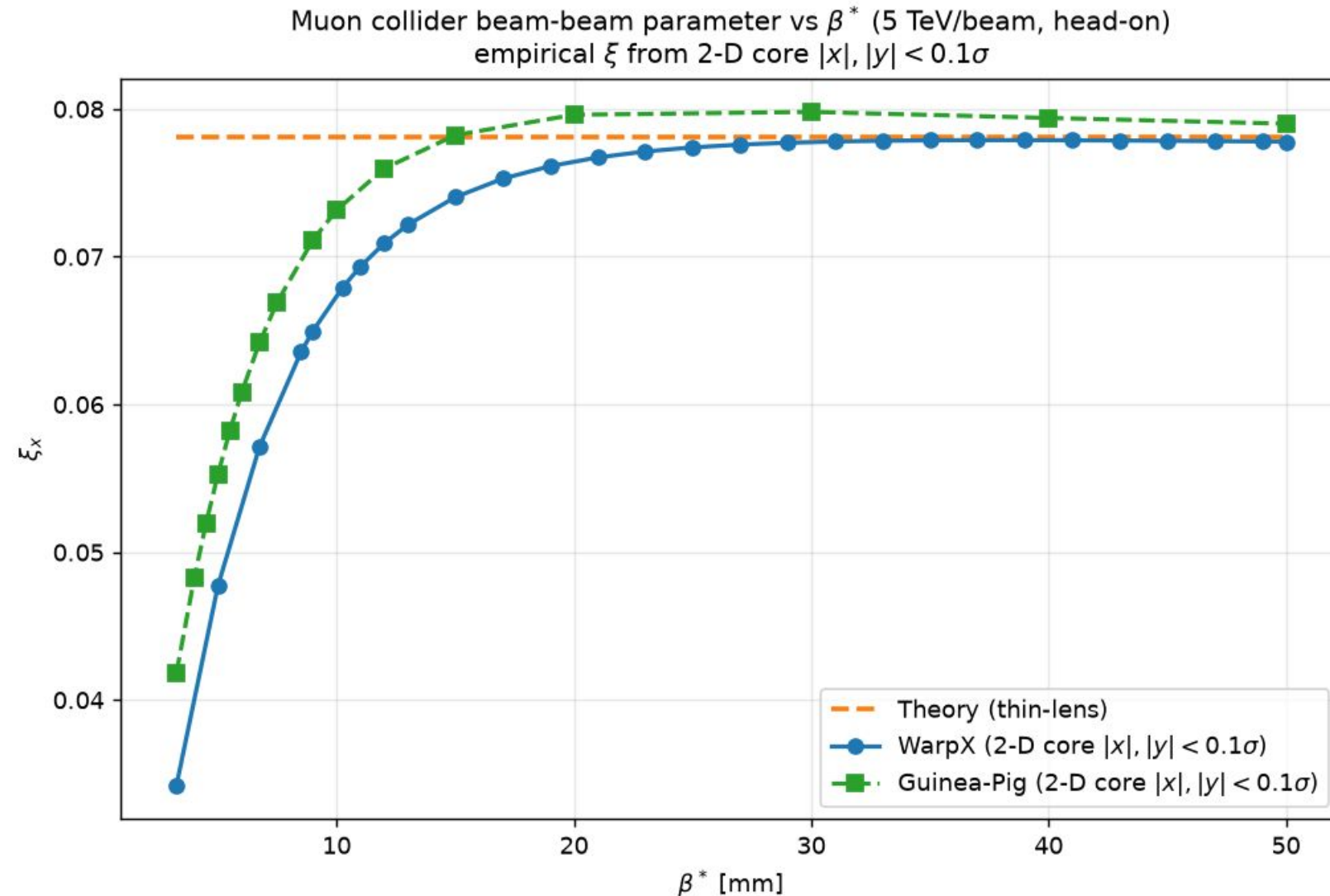
WarpX vs. GP: beam-beam parameter fits

WarpX vs Guinea-Pig: beam-beam kick vs x at IP (x -only $|x| < 0.5\sigma$) — code (columns) $\times$ β^* regime (rows)



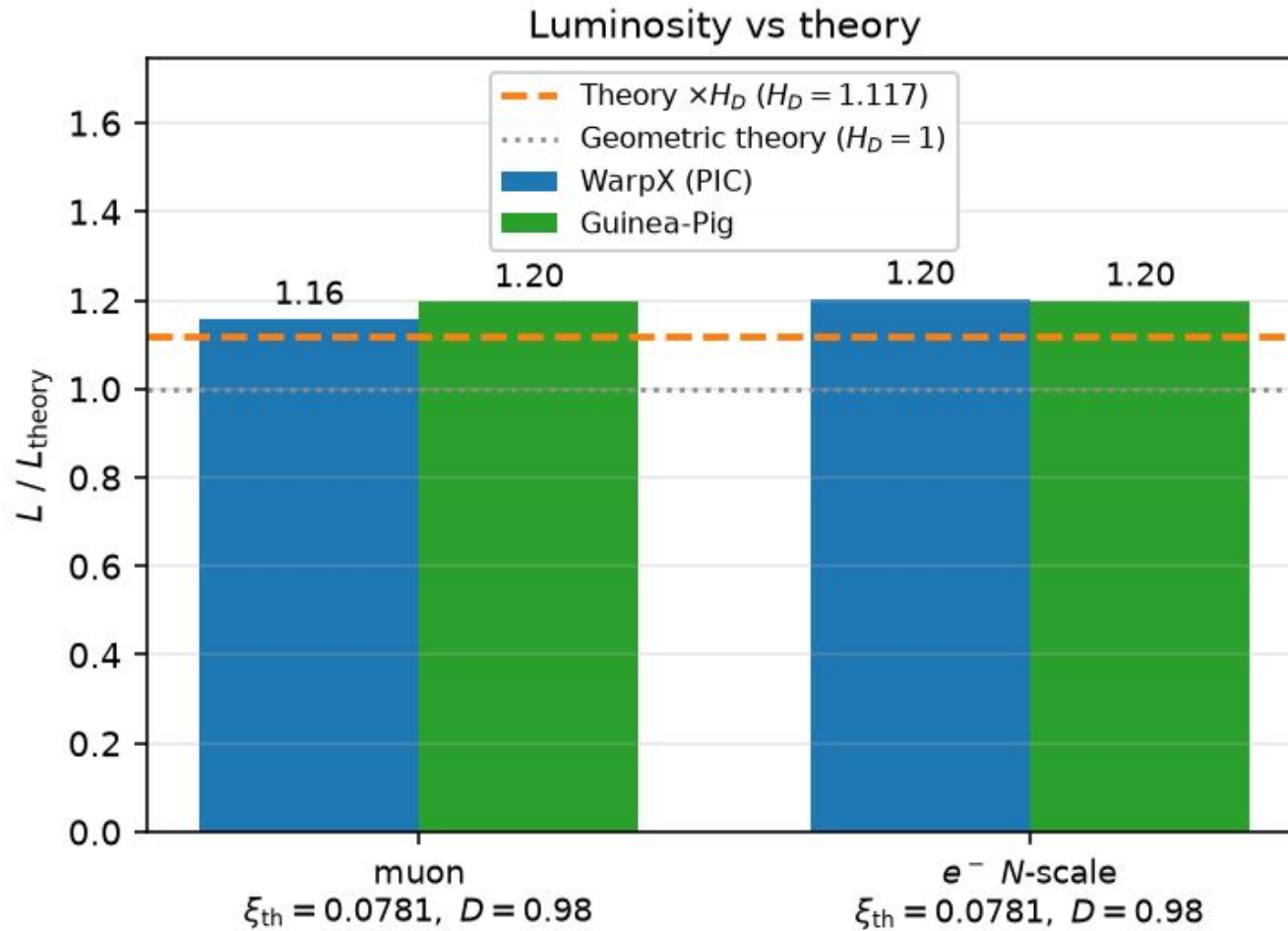
- We want to extract beam-beam parameter from these simulations
 - by fitting the slope of particle position vs. kick
- GP and WarpX dump similar particle data, and achieve very similar beam-beam fits
 - for highly disrupted muon beams (top)
 - and low disruption (bottom)

GP vs. WarpX vs. Thin-lens approximation: Beam-Beam



- GP and WarpX mostly agree on beam-beam parameter
 - both converge to thin-lens prediction at large β^* / low disruption
- Low β^* , as the muon collider specs require, leads to highly disrupted beams

WarpX vs. GP: beam-beam parameter fits



- As a cross-check, I compared WarpX and GP for an 'equivalent' e⁺ e⁻ collision parameters.
 - Analogous disruption and beam-beam parameters accomplished by scaling down the bunch size
 - see appendix for scaling math on mass

Conclusions

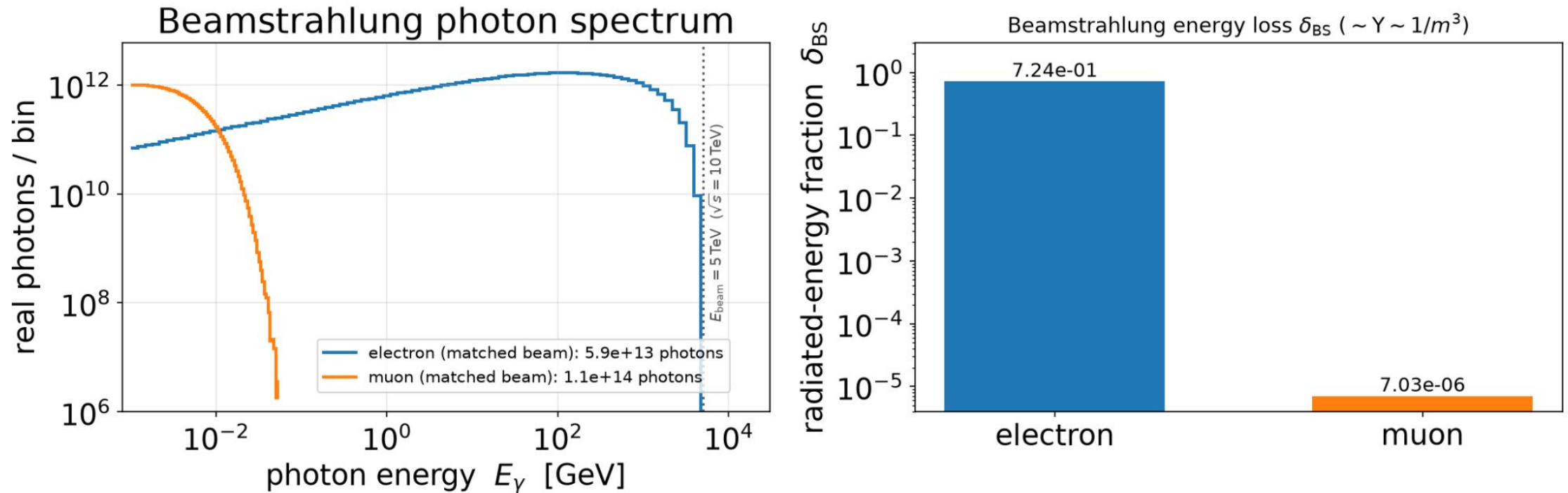
1. GP and WarpX, modified to appropriately handle muons, **agree on muon collider luminosity and beam-beam parameter!**
2. Muon collider luminosities and beam-beam parameter **agree with those obtained by simulating from a comparable e-e collider** with analogous classical parameters (accomplished in this case by scaling the bunch size).
3. The muon collider as designed involves a *highly disrupted beam with non-linear behavior* that is in need of further study.

Next Steps

- In order to do multi-turn analysis (combining WarpX and GP with X-suite), we need more detailed information about the design of the collider ring.
- As of now , we only know about the interaction point.
- We hope you can provide us with a single turn matrix or a full lattice, depending on what is available.

Appendix: Mass suppression of radiation and quantum effects (WarpX)

Quantum-synchrotron suppression: WarpX, identical beams, e vs μ (10 TeV CoM)



measured δ_{BS} suppression $\approx 1.0\text{e}+05$ (lower bound: the electron is quantum-saturated at $\delta_{\text{BS}} \lesssim 1$; the intrinsic Y ratio is $(m_\mu/m_e)^3 = 8.8 \times 10^6$)

Appendix: Scaling of classical beam parameters with Beta*

$$\sigma_0 = \sqrt{\frac{\varepsilon_N \beta^*}{\gamma}} \propto \sqrt{\beta^*}, \quad \sigma_0^2 \propto \beta^*$$

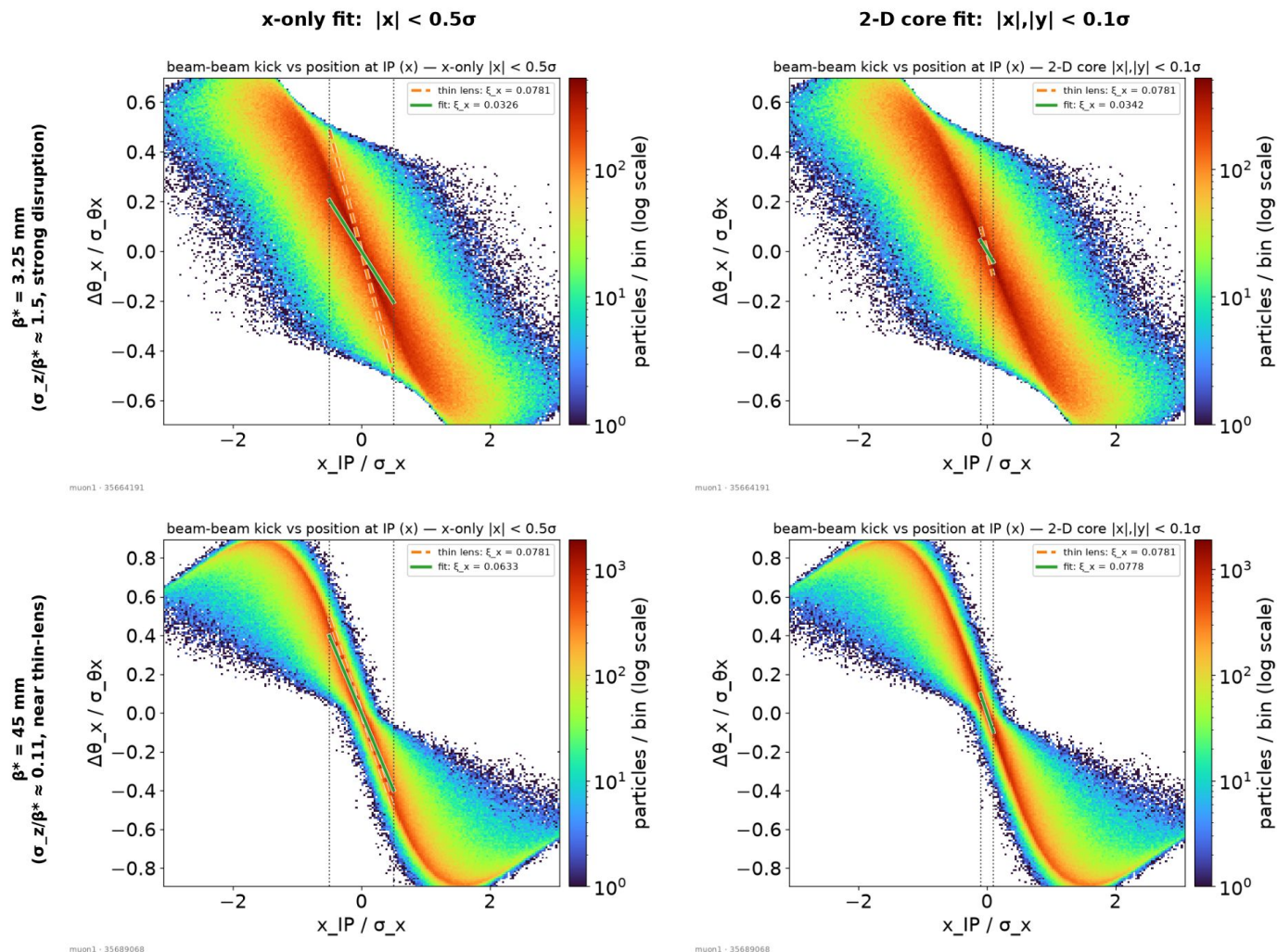
$$\mathcal{L} = \frac{N_1 N_2}{4\pi\sigma_0^2} F_{\text{hg}}\left(\frac{\beta^*}{\sigma_z}\right) H_D(D(\beta^*)) \propto \frac{1}{\beta^*} F_{\text{hg}}\left(\frac{\beta^*}{\sigma_z}\right) H_D(D(\beta^*))$$

$$D = \frac{rN\sigma_z}{\gamma\sigma_0^2} = \frac{rN\sigma_z}{\varepsilon_N\beta^*} \propto \frac{1}{\beta^*},$$

$$\xi = \frac{rN\beta^*}{4\pi\gamma\sigma_0^2} = \frac{rN}{4\pi\varepsilon_N} \propto (\beta^*)^0.$$

Appendix: beam-beam parameter fits with diff fitting procedures

WarpX: beam-beam phase-space density (kick vs x at IP) — fit window (columns) $\times$ β^* regime (rows)



- For a round beam, fitting the core
 - 'Analogous disruption and beam-beam parameters accomplished by scaling down the bunch size.
 - see appendix for scaling math

Appendix: Mass scaling of classical beam parameters

$$r \propto \frac{1}{m}, \quad \gamma \propto \frac{1}{m}, \quad \sigma_0 \propto \sqrt{m}$$

$$\mathcal{L} = \frac{N_1 N_2}{4\pi\sigma_0^2} H_D \propto \frac{1}{m}$$

$$D = \frac{r N \sigma_z}{\gamma \sigma_0^2} = \frac{e^2 N \sigma_z}{4\pi\epsilon_0 E \sigma_0^2} \propto \frac{1}{m}$$

$$\xi = \frac{r N \beta^*}{4\pi\gamma\sigma_0^2} = \frac{e^2 N \beta^*}{16\pi^2\epsilon_0 E \sigma_0^2} \propto \frac{1}{m}$$

- Holding fixed emittance and beam size, classical beam parameters scale linearly with the mass of particle.
- Therefore, in order to do a 1-1 comparison of the muon collider vs. e collider, it's necessary to 'scale' at least one of the parameters.
 - For now, we try just scaling N to compensate.

Appendix: Muon Collider Input Parameters (GP and WarpX)

#	Parameter	Unit	Table 6.1 relaxed target	WarpX (muon1_input.txt)	GUINEA-PIG (gp_template.acc)
1	Center of mass energy	TeV	10	energy = 5*TeV (CoM = 2x)	energy=5000 (GeV, CoM = 2x)
2	Geometric luminosity	$10^{34} \text{ cm}^{-2}\text{s}^{-1}$	5.77 19.2	OUTPUT (ColliderRelevant, DiffLumi_beam1_beam2)	OUTPUT (do_lumi / num_lumi; here do_lumi=0)
3	Beam energy	TeV	5	energy = 5*TeV	energy=5000 (GeV)
4	Relativistic Lorentz factor	-	47322	gamma = energy/mc2, mc2 = m_mu*c^2 [correct]	gamma_beam = ebeam/EMASS [electron mass -> WRONG]
5	Circumference	km	$\approx$ 10	—	—
6	Dist. of last magnet to IP	m	6	—	—
7	Repetition rate	Hz	5	— (no rep-rate concept)	f_rep=1.0 (Hz) [$\neq$ 5]
8	Bunch intensity (1 bunch/beam)	10^{12}	1.80	npart = 1.80e12 (charge = q_e*npart)	particles=180.0 (x1e10), n_b=1
9	Injected beam power per beam	MW	7.2	—	—
10	Normalized transverse rms emittance	μm	25	emit_n = 25*micro (emitx_n = emity_n)	emitt_x=emitt_y=25.0 [muon-patched build]
11	Longitudinal norm. rms emittance	eVs	0.025	implied by sigma_z x espread	implied by sigma_z x espread
12	Relative rms momentum spread	10^{-3}	0.3 1	espread = 0.3e-3 * gamma (rel. = 0.3e-3)	espread=0.0003 (relative)
13	RMS bunch length in space	mm	5 1.5	sigmaz = 5*milli	sigma_z=5000.0 (μm)
14	RMS bunch length in time domain	ns	.017 0.005	derived = sigmaz/c	derived = sigma_z/c
15	Twiss betatron function at the IP	mm	5 1.5	beta = 5*milli (betax = betay)	beta_x=beta_y=5.0 (mm)
16	Energy loss per turn	MeV	$\approx$ 27.2	—	—
17	Integrated RF gradient	MV	30	—	—

Direct 1:1 input
Derived, not an input
Output, not an input
No correspondence
Patched build required
Mismatch / divergence

Appendix: WarpX muon-patches

File (everything in subdirectory Source/Particles/ElementaryProcess/)	Description of change
QEDInternals/QedChiFunctions.H	Add mass scaling in computation of Schwinger field χ : $\chi \propto (m / m_e)^3$
QEDInternals/QuantumSyncEngineWrapper.H	Propagate input mass through everything that calls χ
Input file change	Numerical integrals of $T(\chi)$ [a modified Bessel function] normally has a pre-computed table; Since χ can now range over orders of magnitude, we have to modify the range over which $T(\chi)$ is computed.
QEDPhotonEmission.H	WarpX 'cleans' low energy photons by dropping photons under a certain energy, computed using the mass of the electron. This threshold needs to be lowered for heavier lepton.
PICSAR [Particle In Cell Scalable Application Resources] dependency: quantum_sync_engine_core.hpp, gamma_functions.hpp	PICSAR library is used to compute dN/dt [# photons/time] and hard codes the electron mass, so this has to be updated to pass through as well.