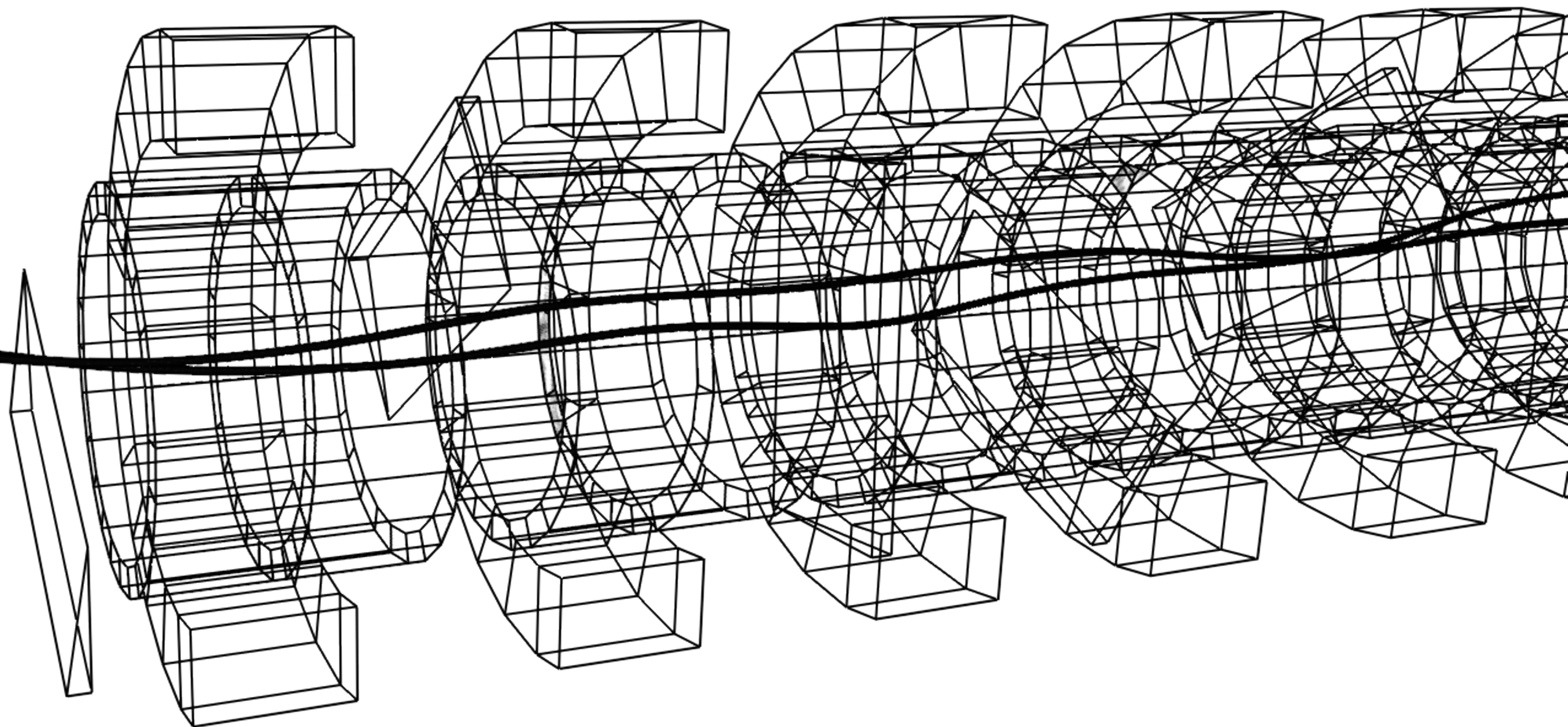


Summary of HF0F0 Studies

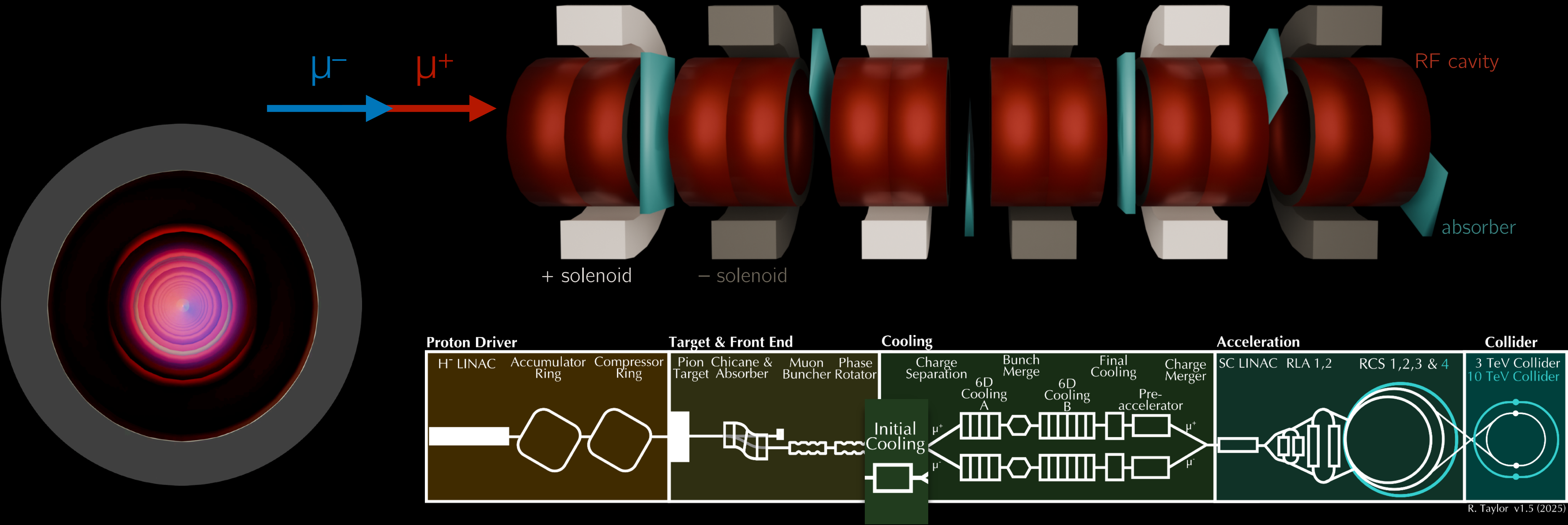
μCATALYST Summer 2026

Caroline Riggall
September 14, 2026



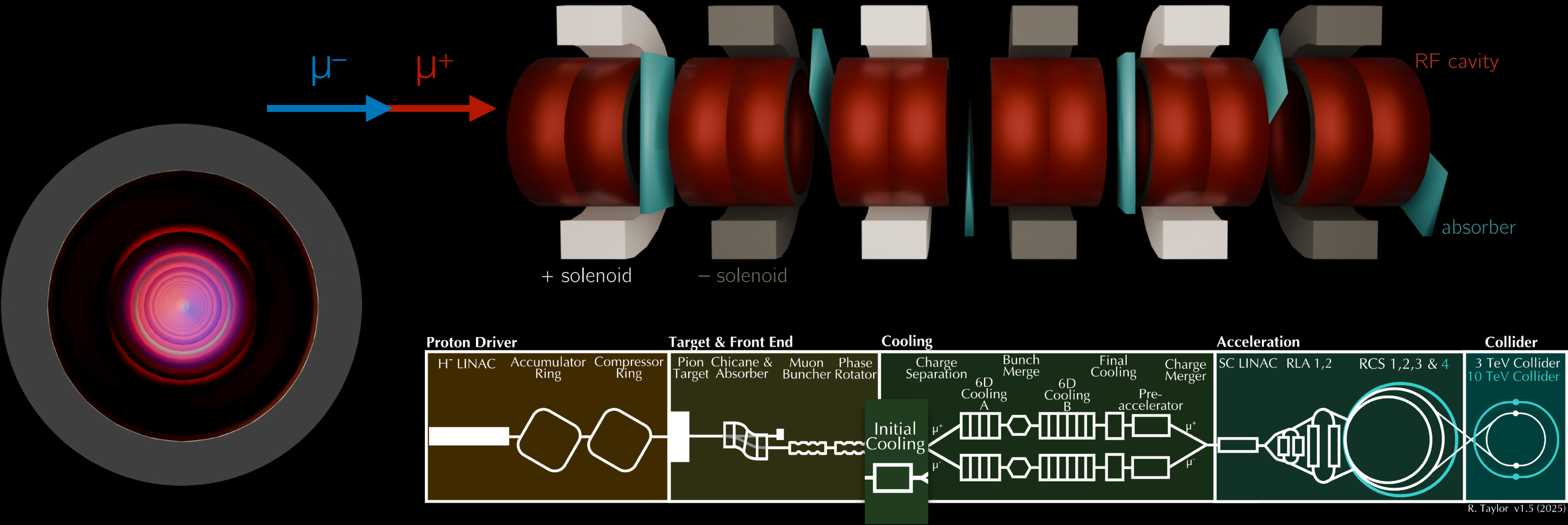
Recap: what (and why) is HF0F0?

The Helical FOFO Snake (HF0FO) is a design for 6D cooling of μ^+ and μ^- in a single channel, enabling improved transmission rate during charge separation and reduced construction costs for a muon collider.



Recap: what (and why) is HF0F0?

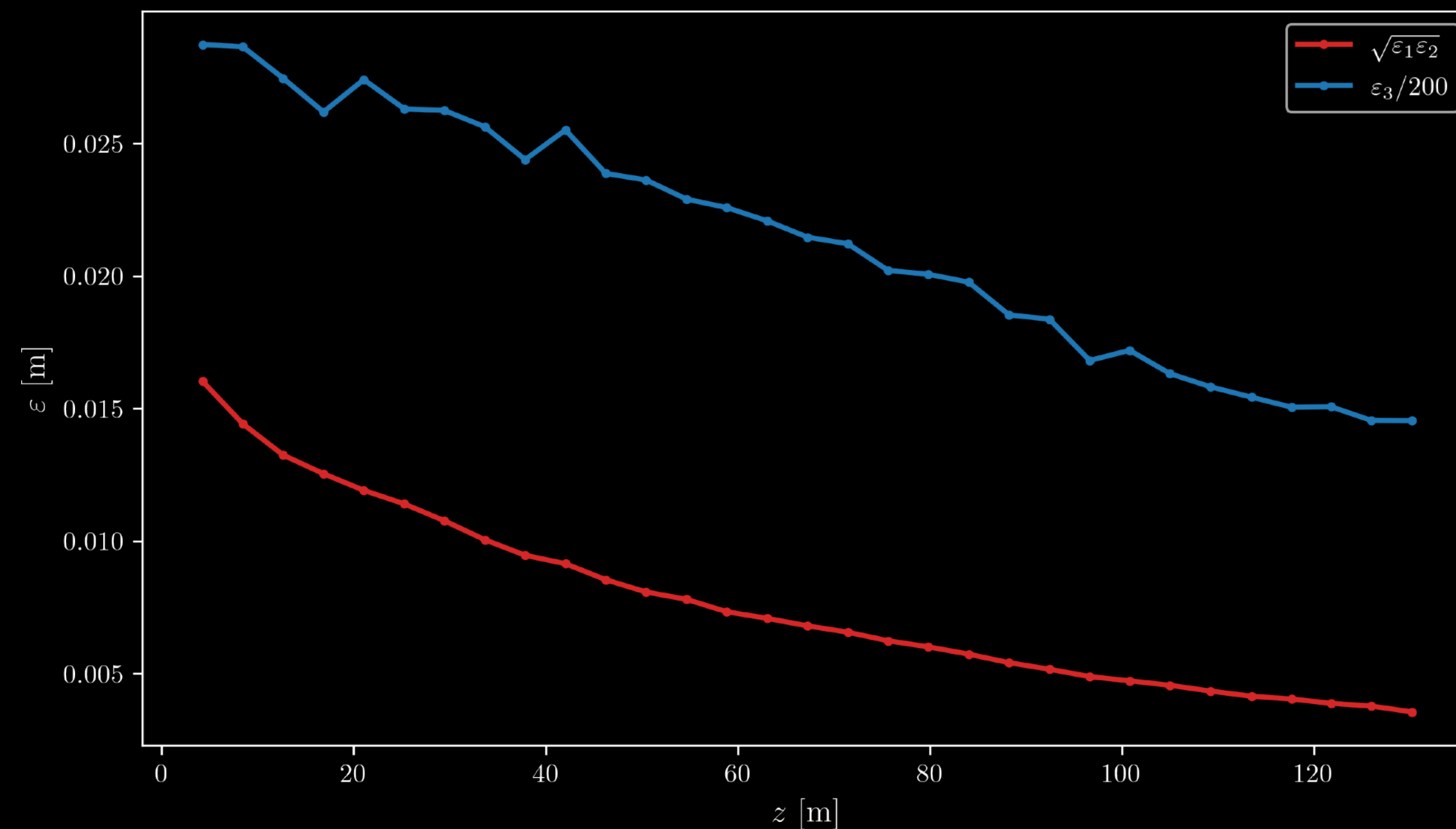
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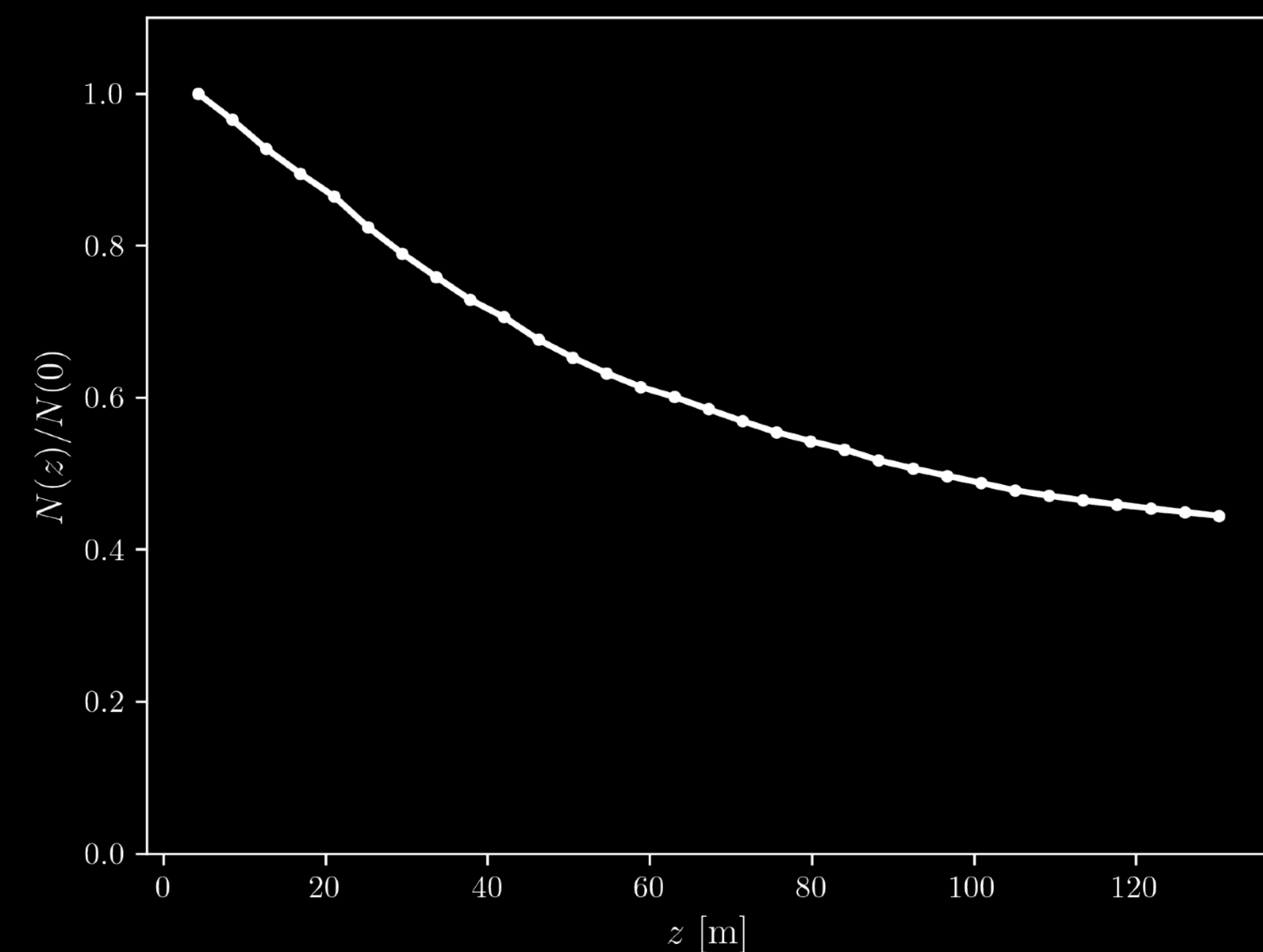
Recap: status of HF0F0 & my goals for the summer

We observe an order of magnitude in emittance reduction with a beam transmission rate of 67% in the current design.

Eigenmode emittances



Transmission rate



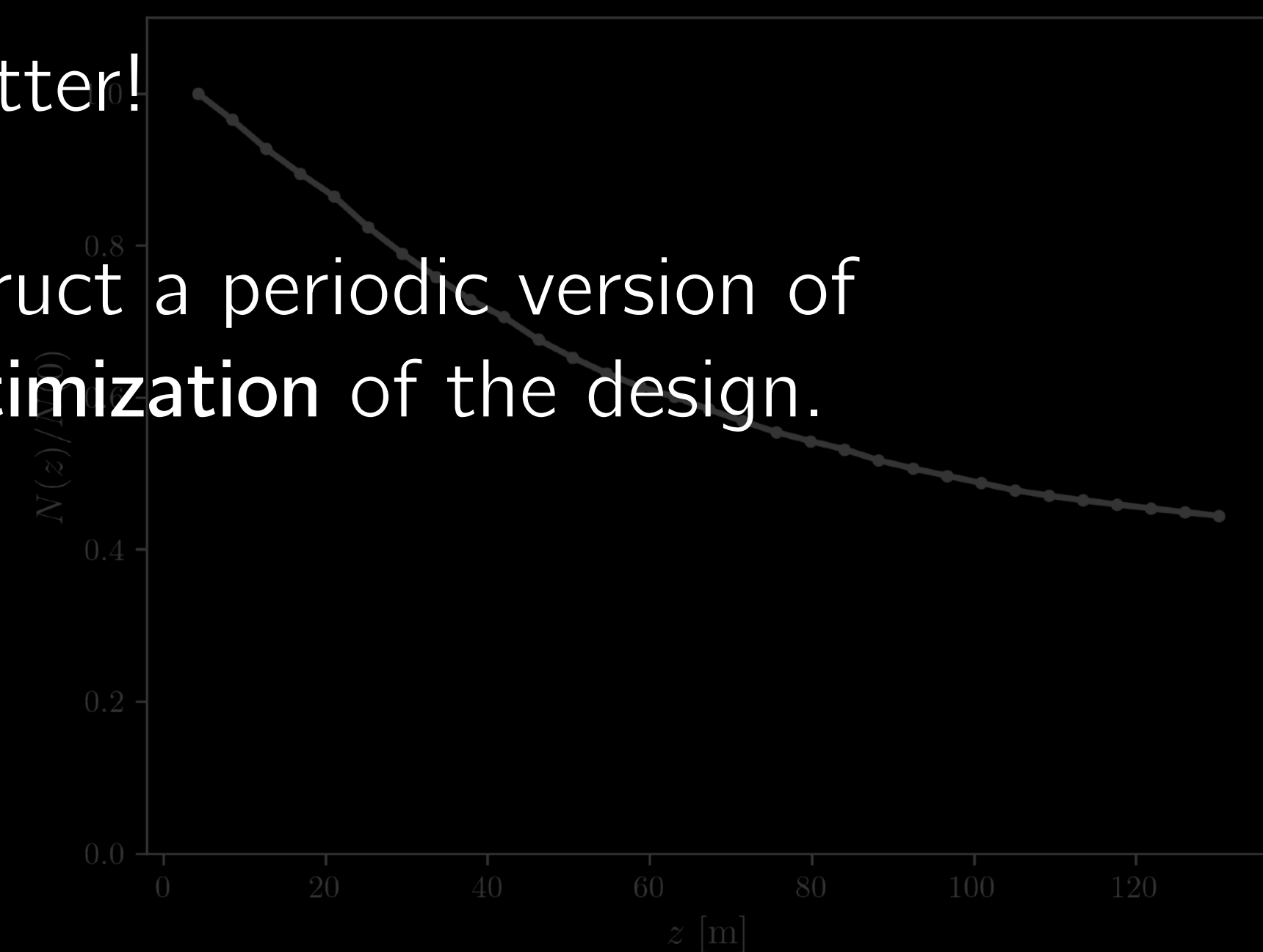
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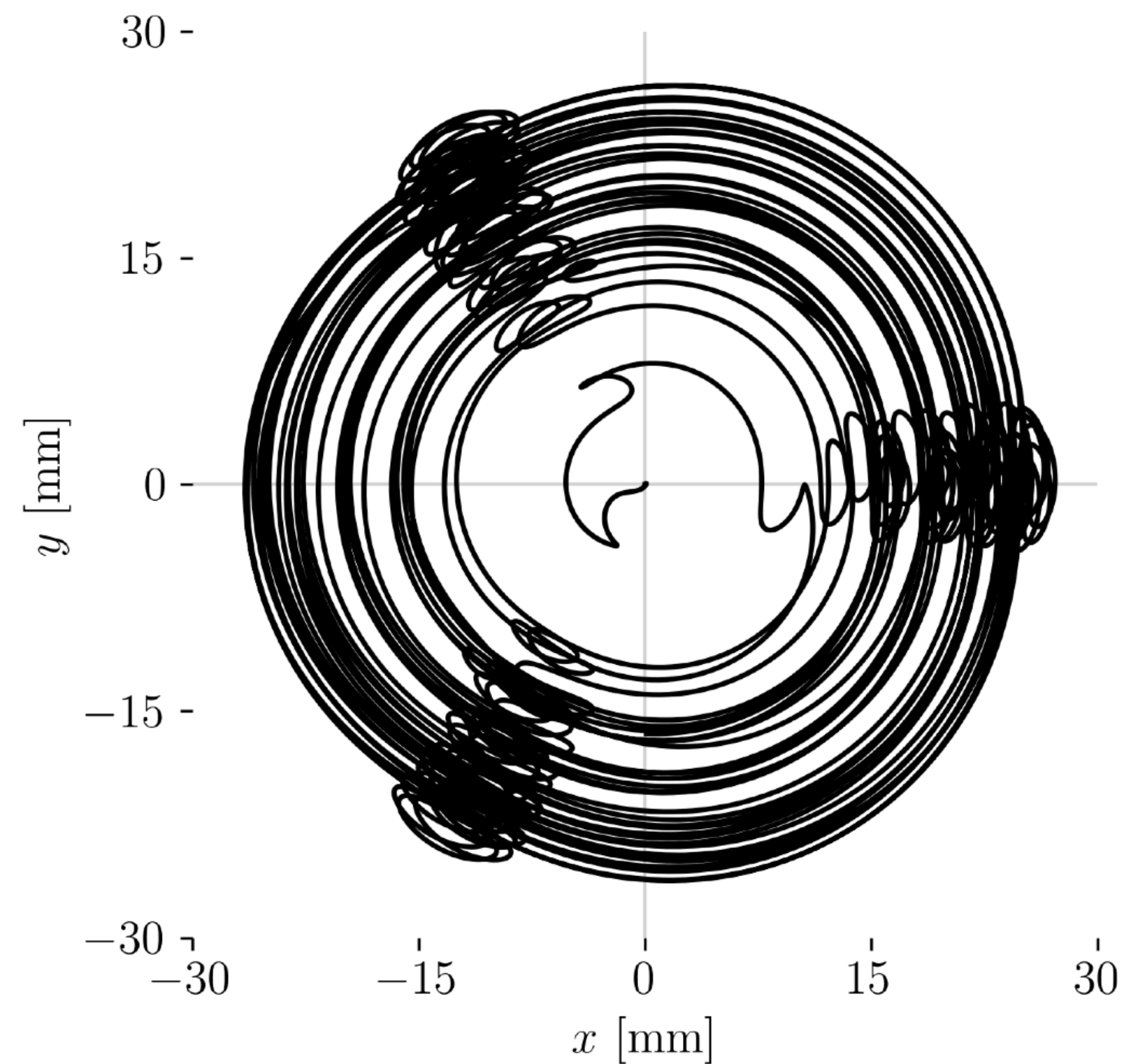


Transmission rate

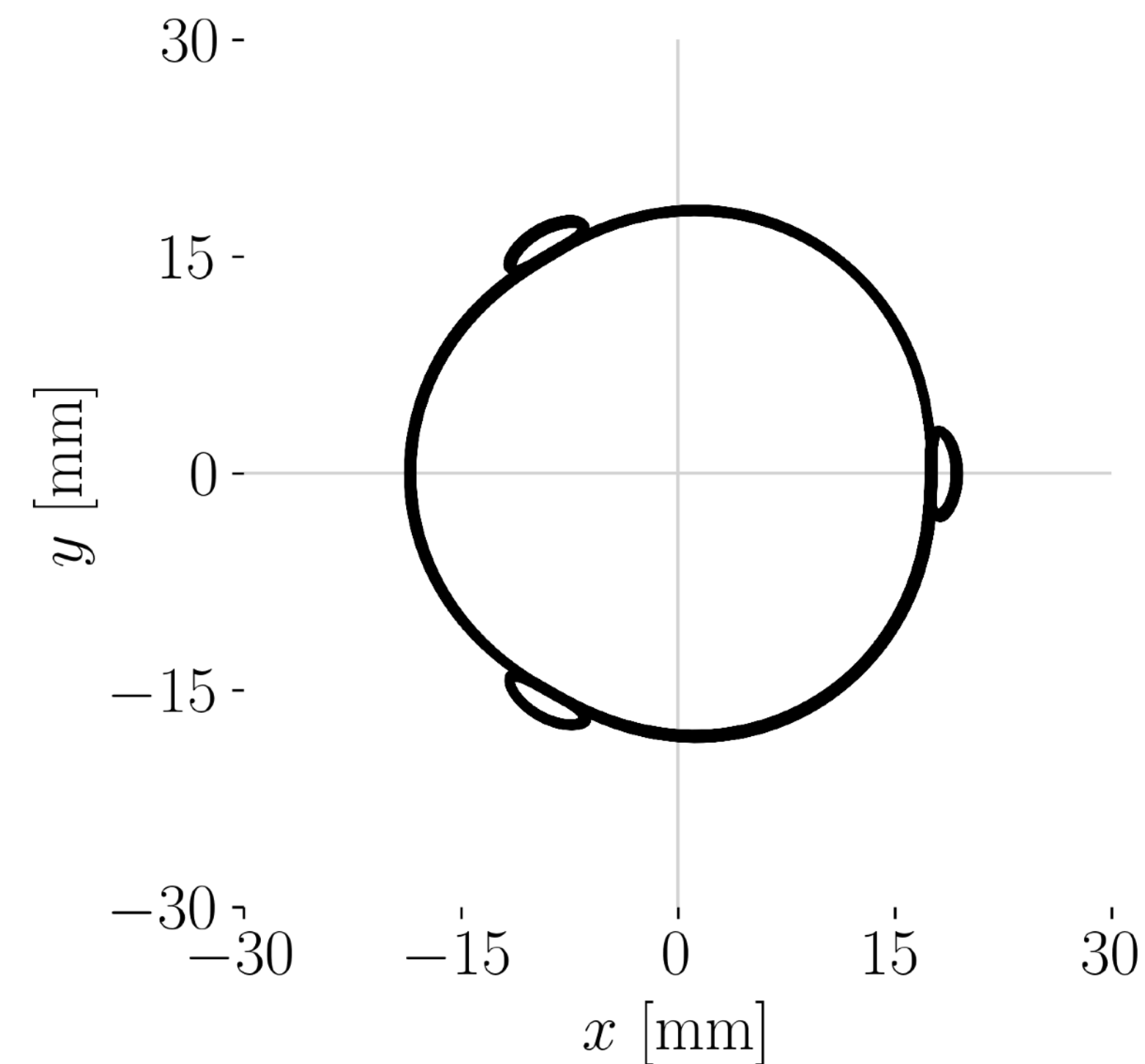


Toward a periodic design – motivation

The baseline design:

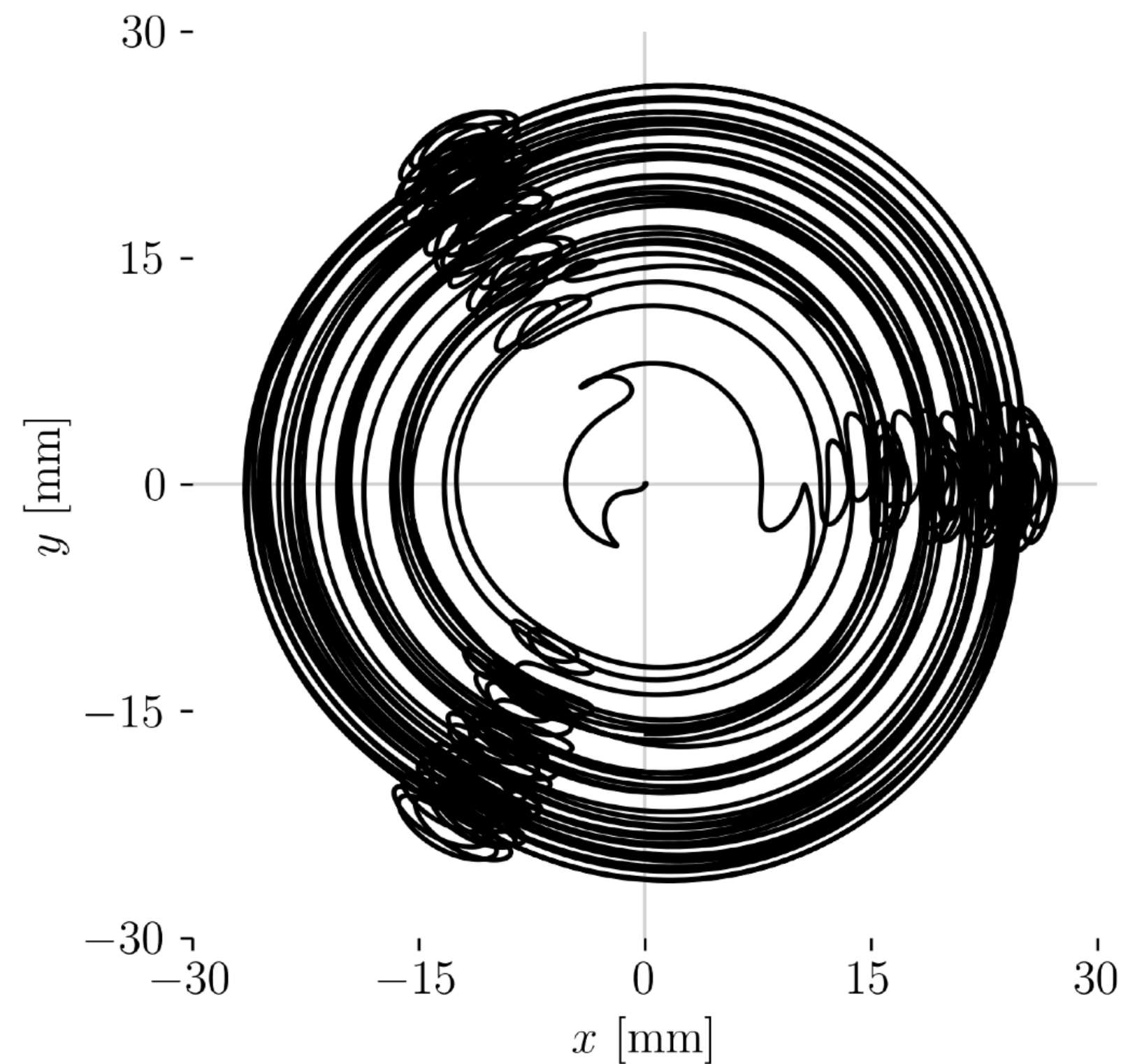


What I had at the start of the summer:

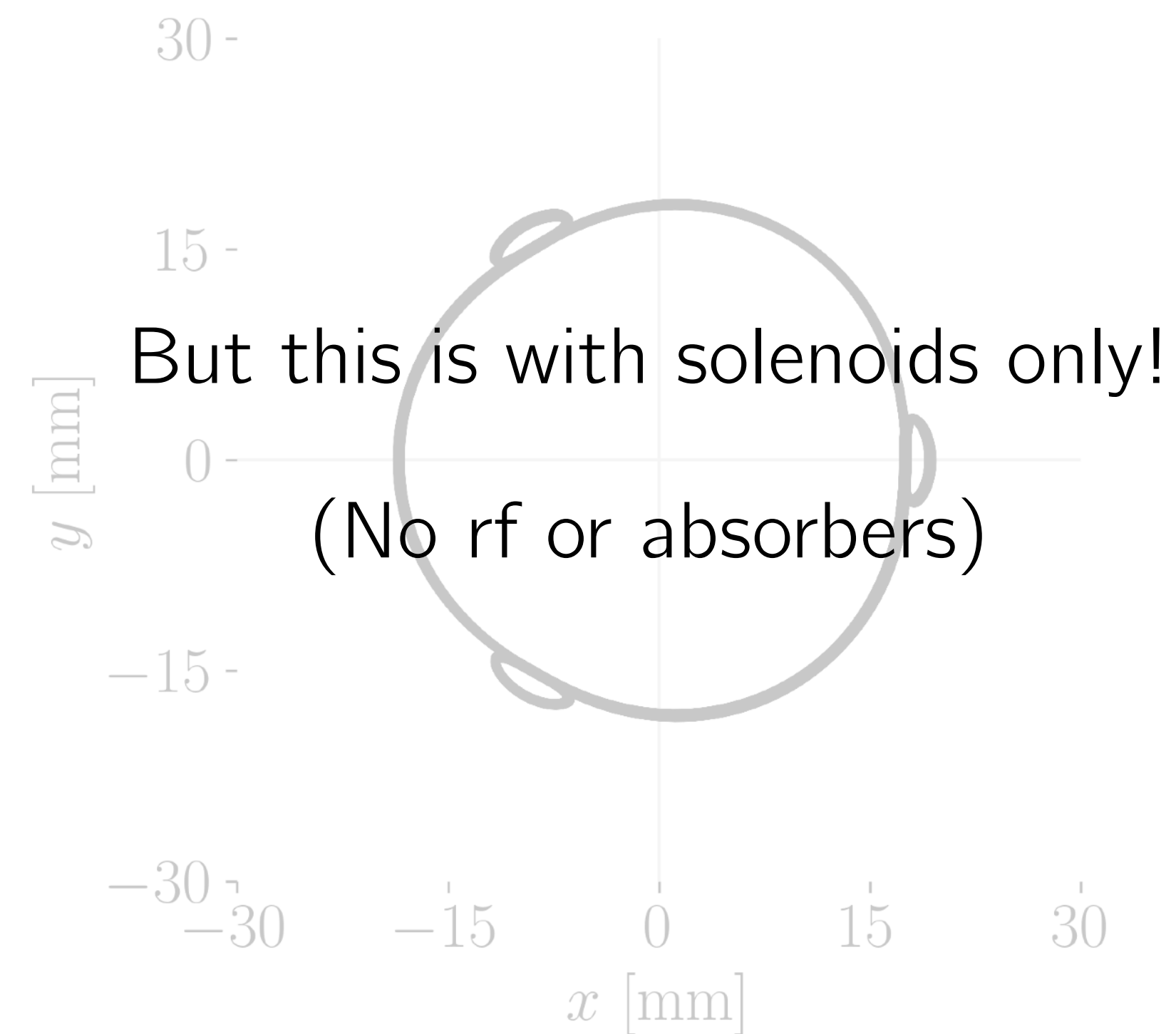


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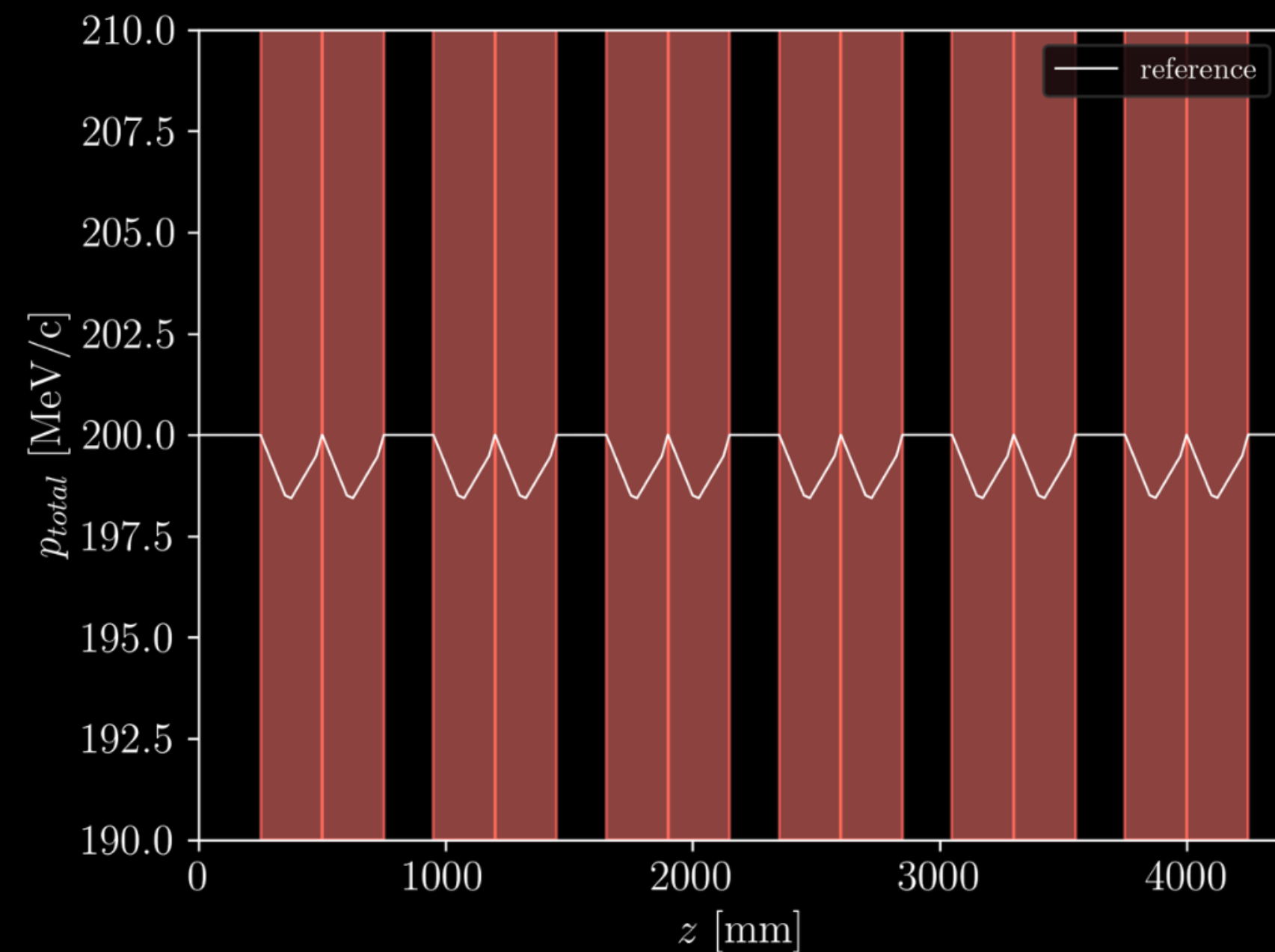
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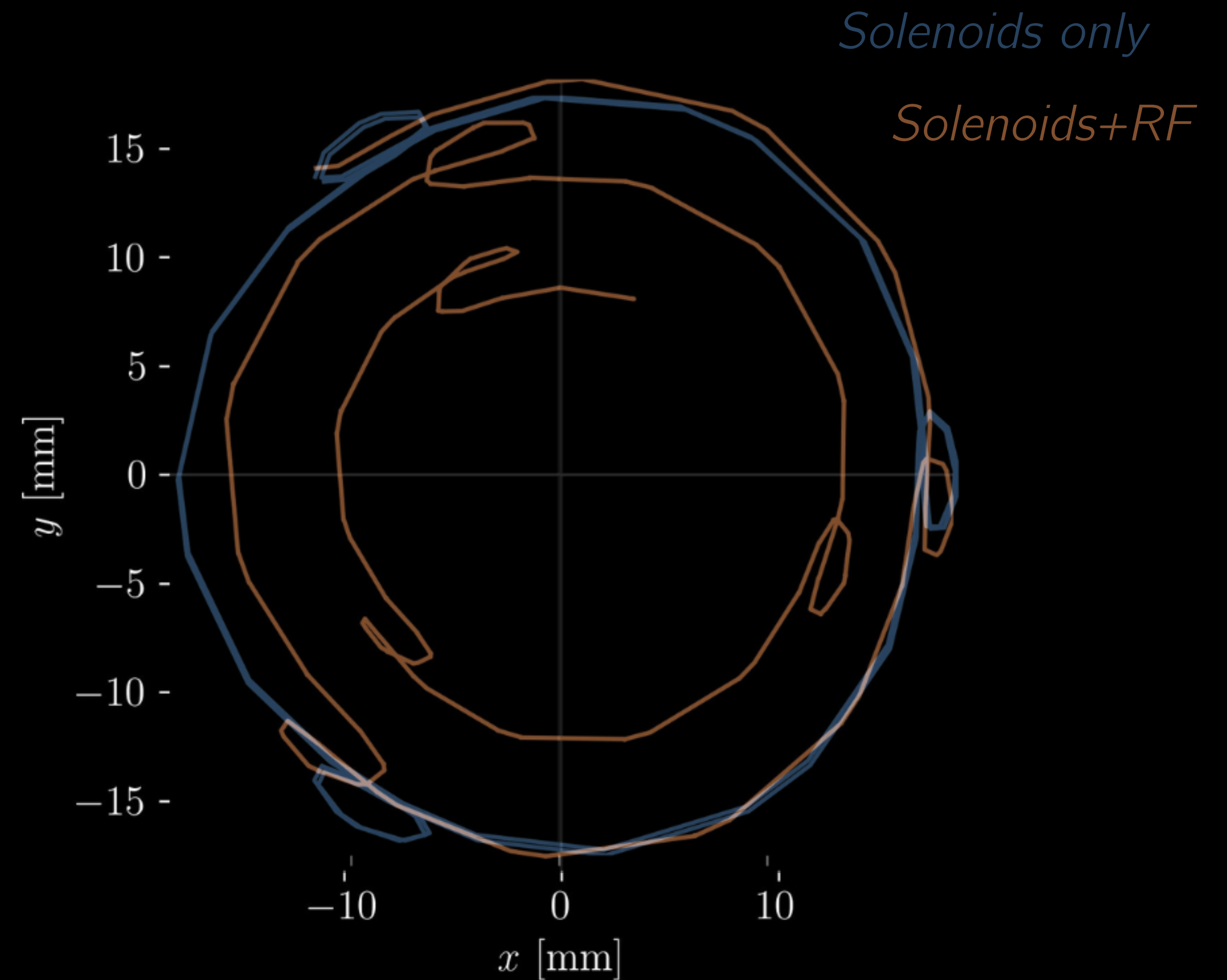
What I had at the start of the summer:



Toward a periodic design – methodology



Adding rf w/ no net acceleration to periodic optics design perturbs periodic orbit!



Toward a periodic design – methodology

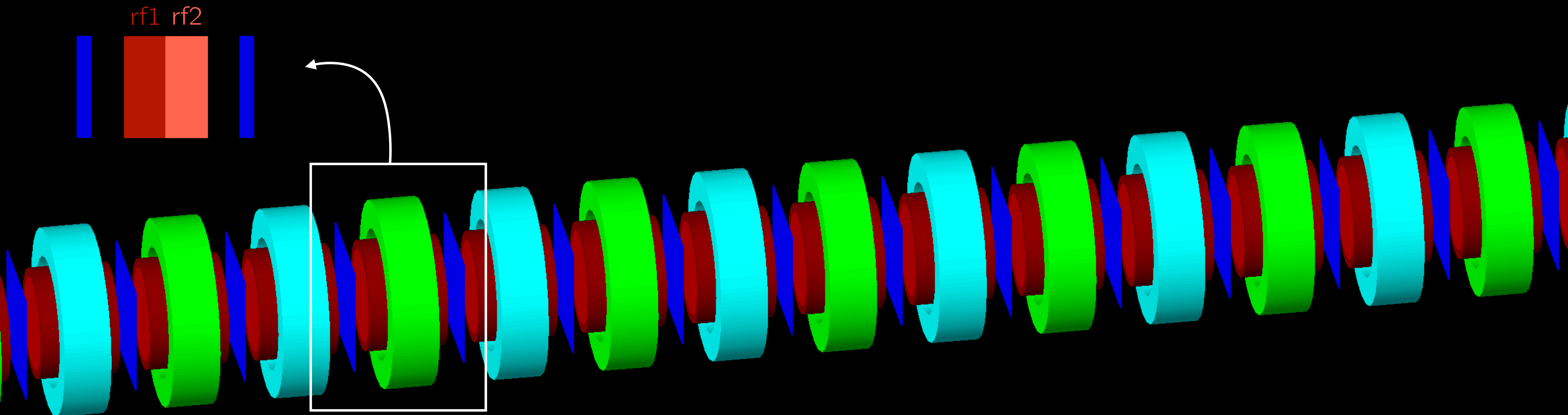
Add in small amount of
absorber



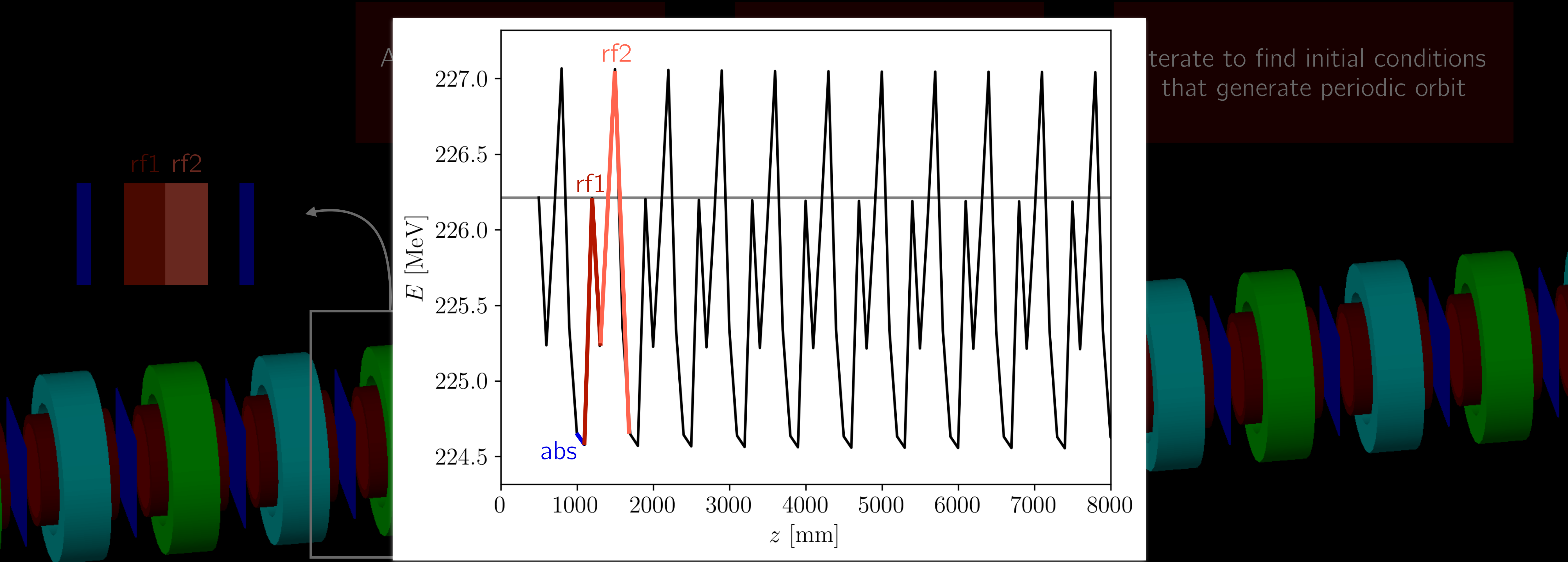
Tune RF timing until
energy vs. z is periodic



Iterate to find initial conditions
that generate periodic orbit

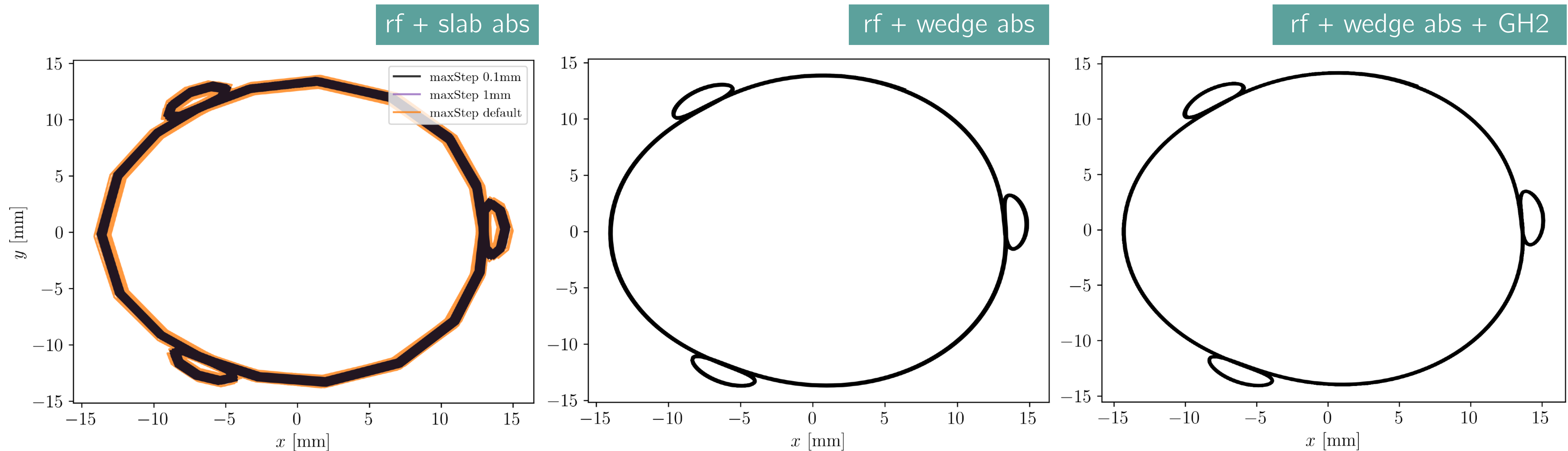


Toward a periodic design – methodology



Toward a periodic design – results

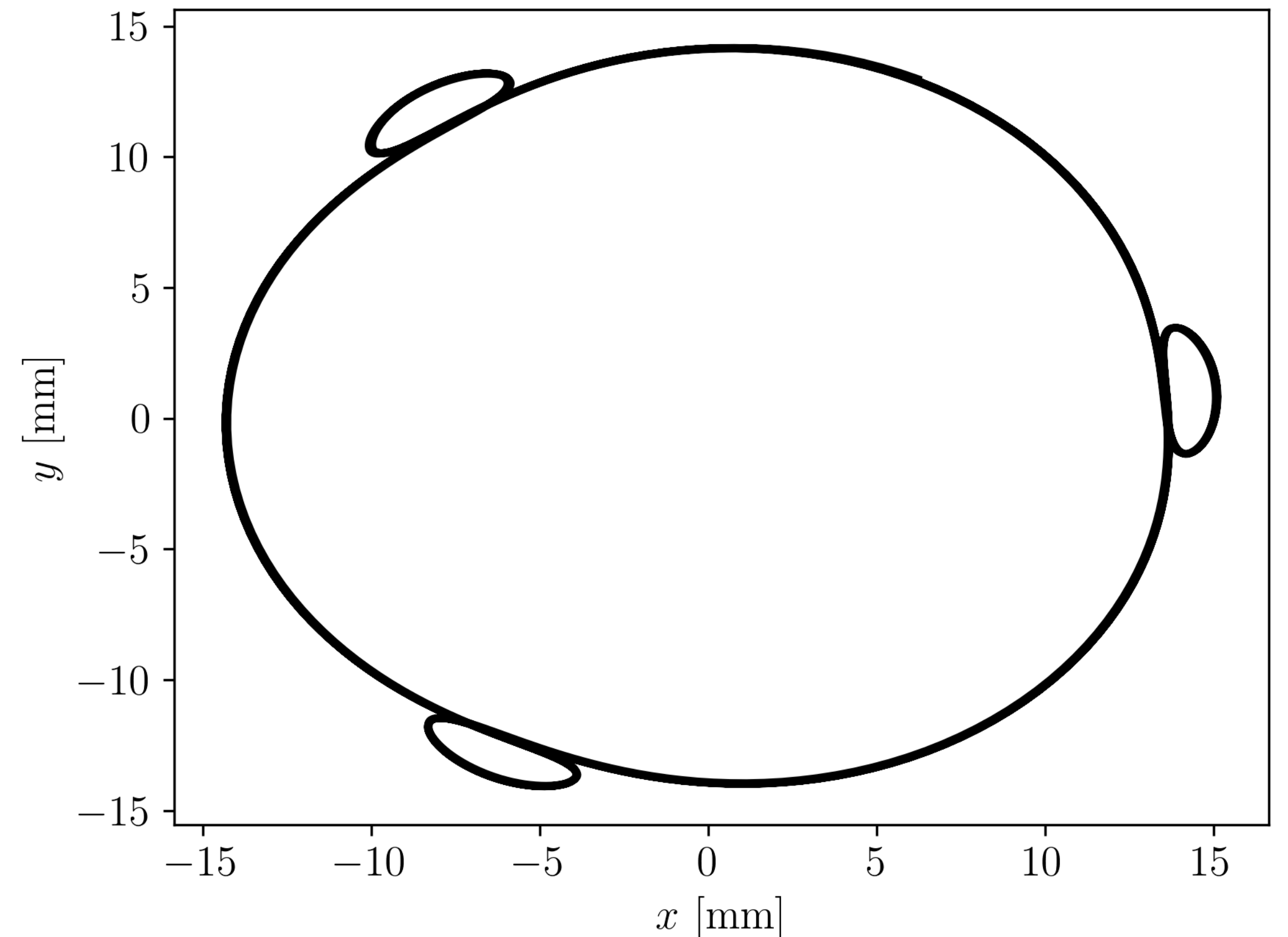
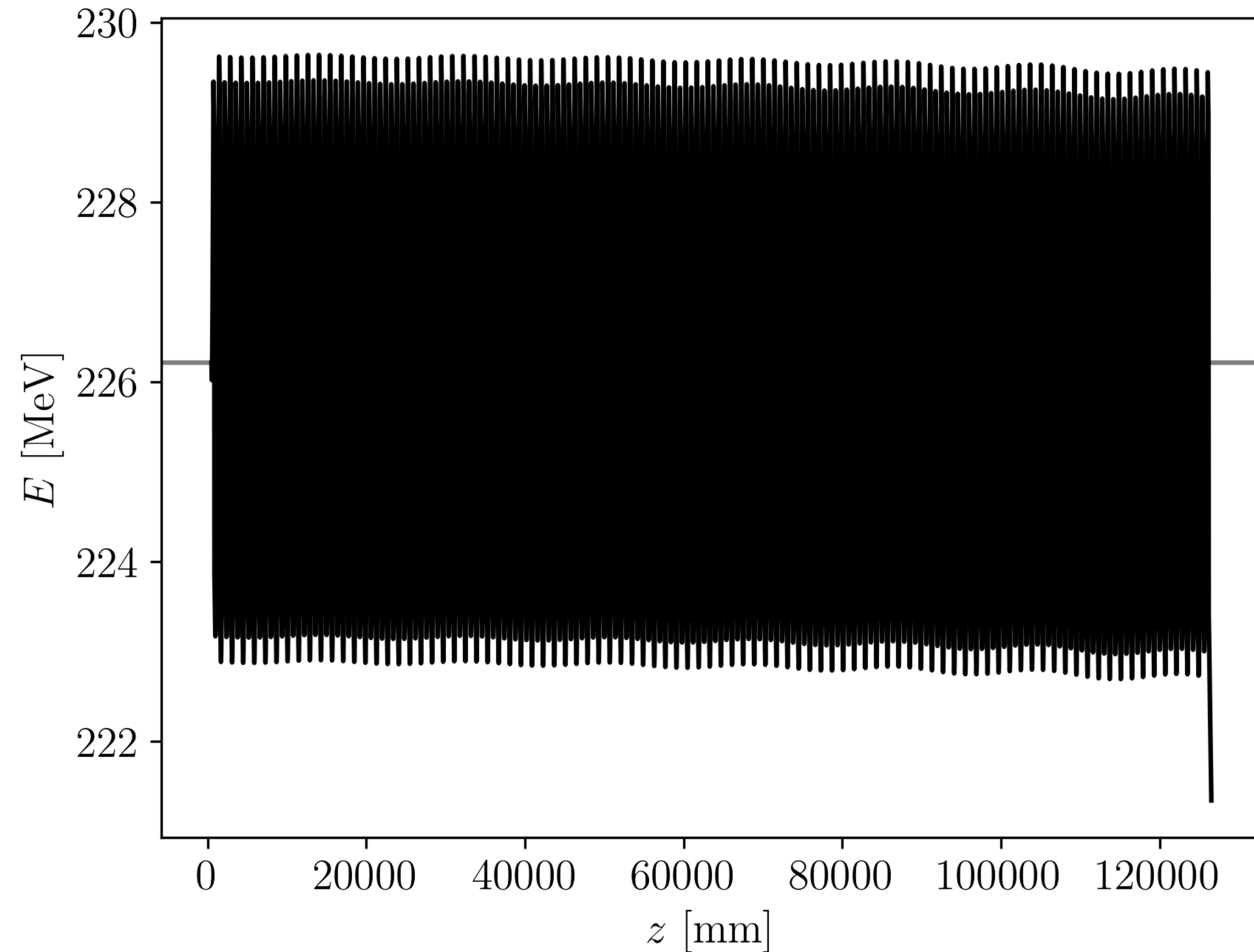
I made a lot of versions of the lattice...



...which we can use for various comparison studies!

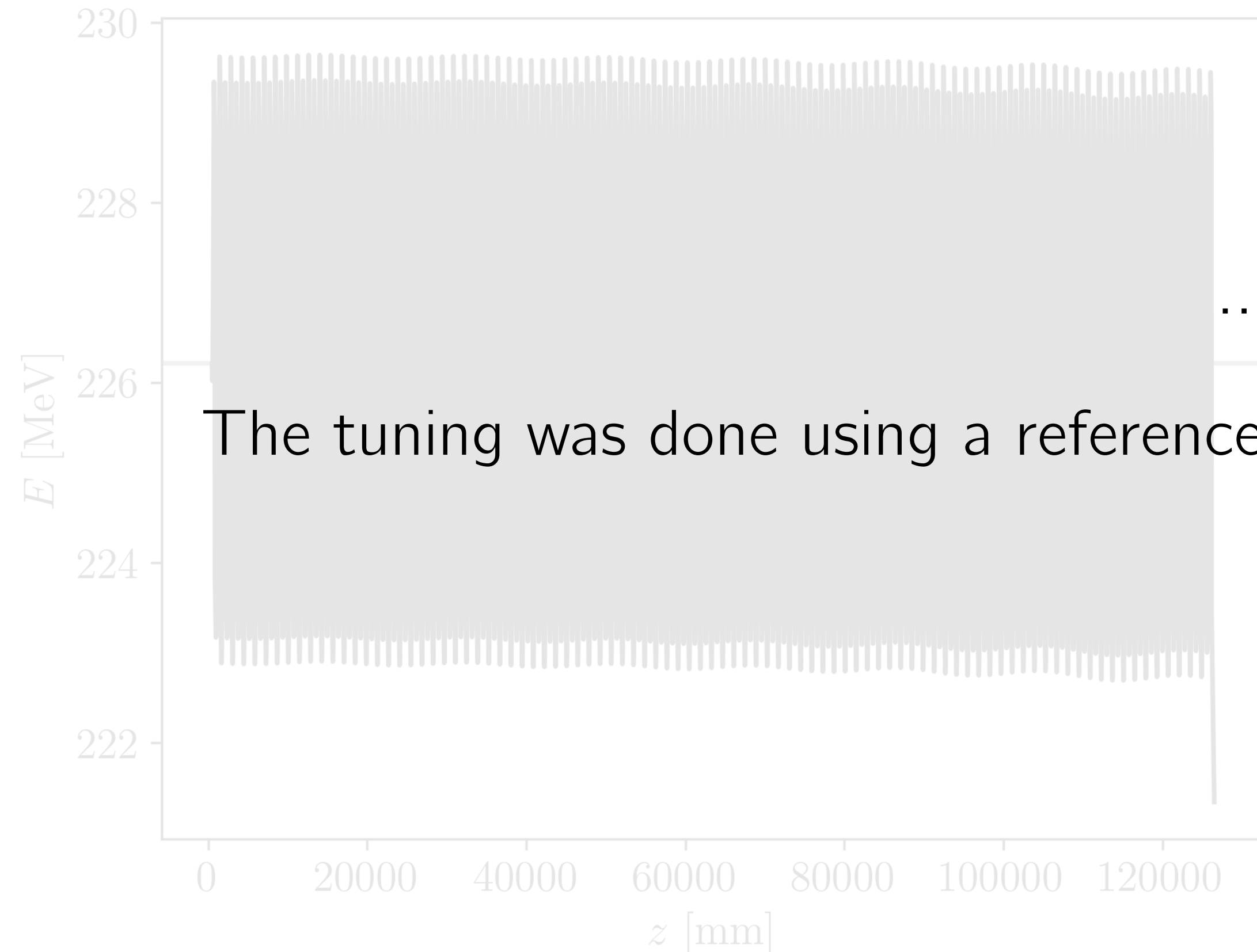
Toward a periodic design – conclusions

We now have a periodic design which operates at (or rather oscillates about) a constant momentum of 200 MeV/c



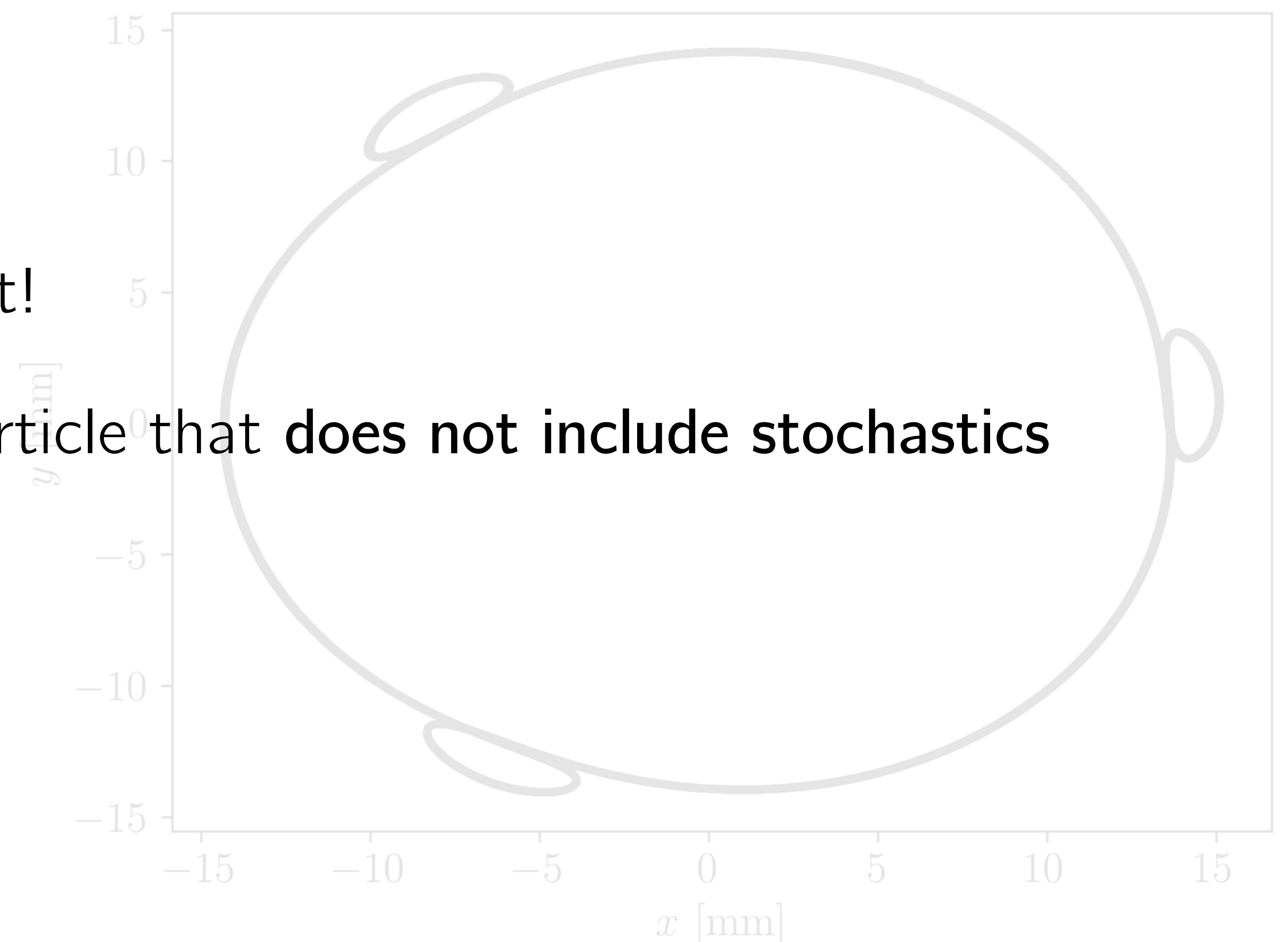
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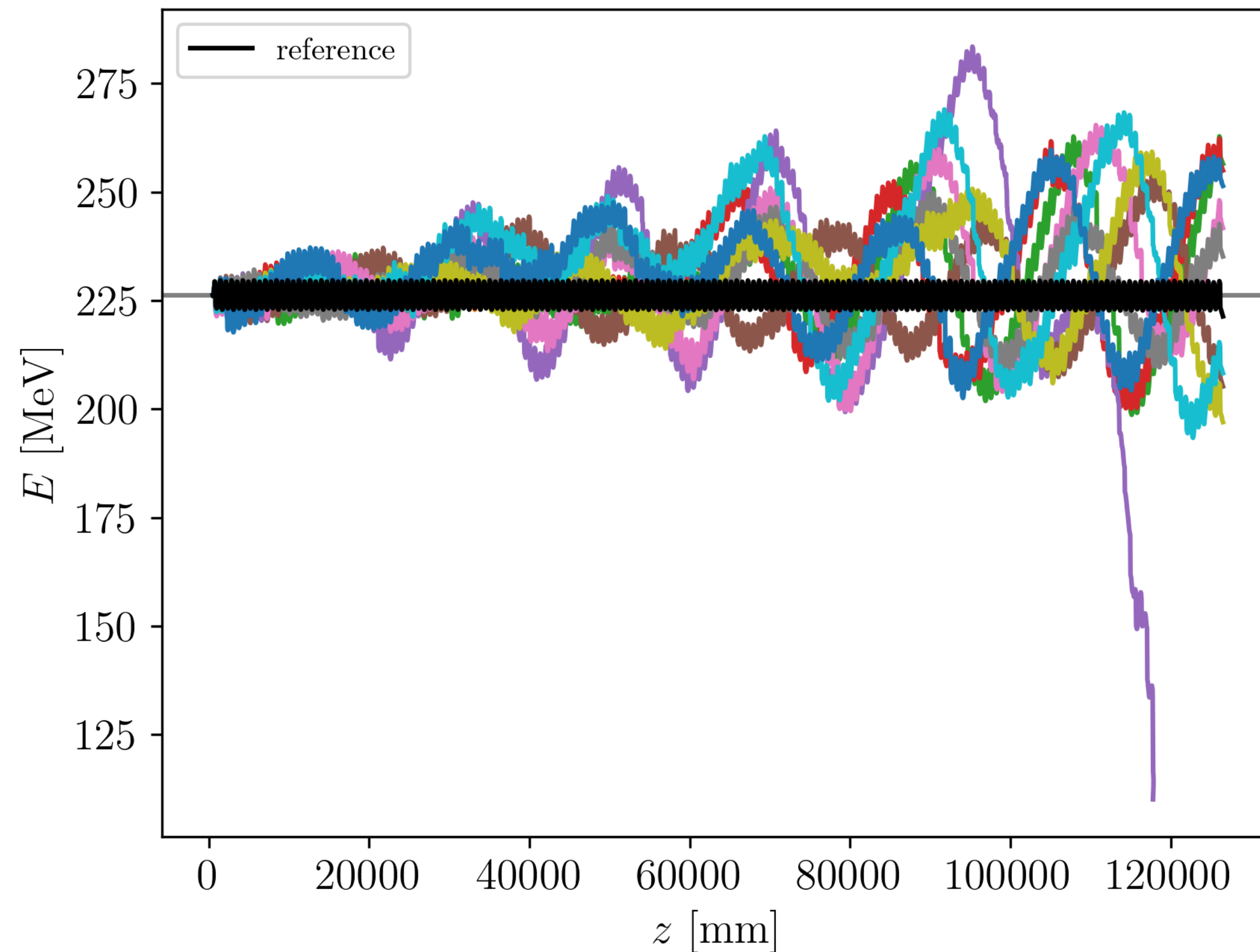
...But!

The tuning was done using a reference particle that **does not include stochastics**

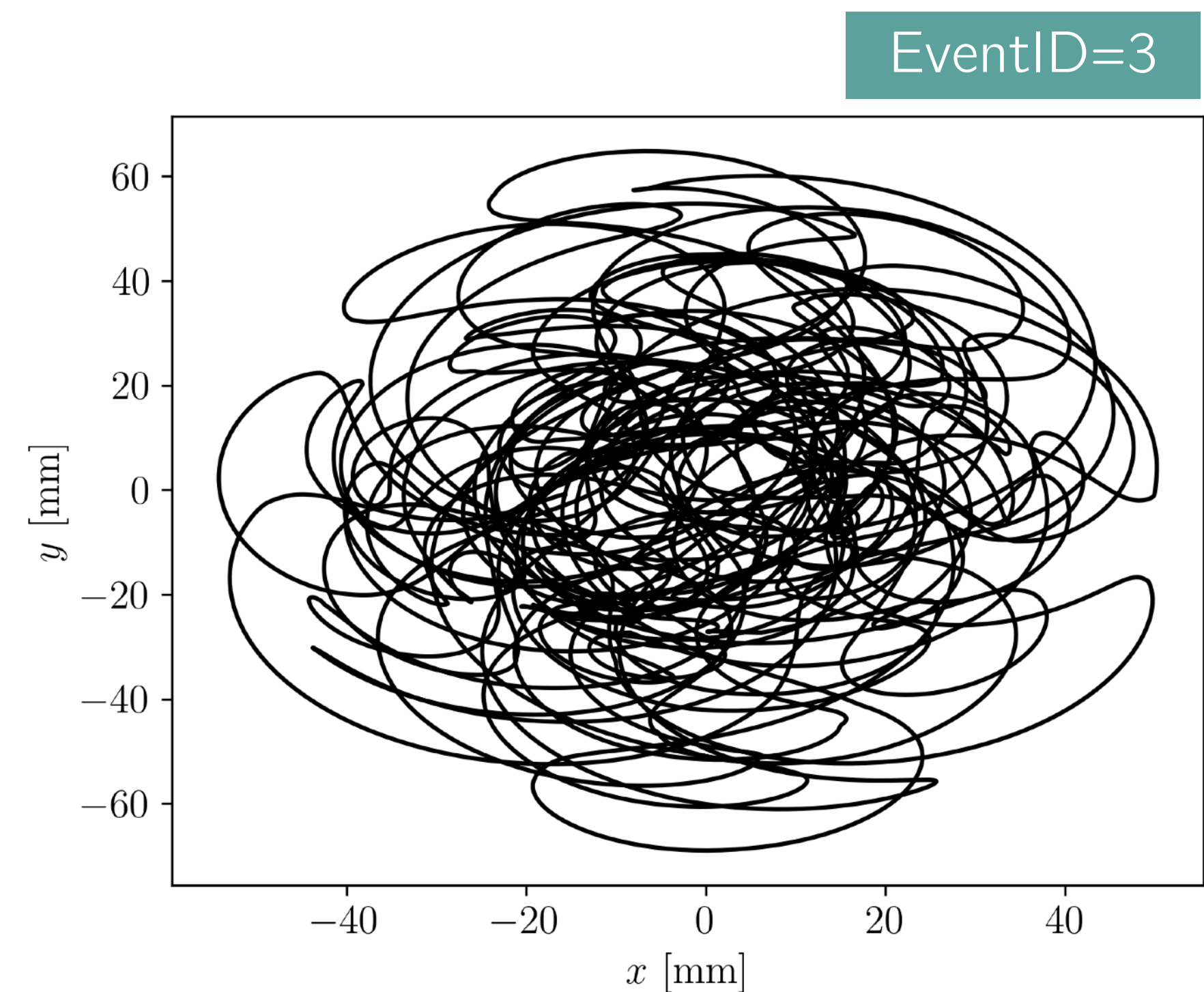


Toward a periodic design – next steps

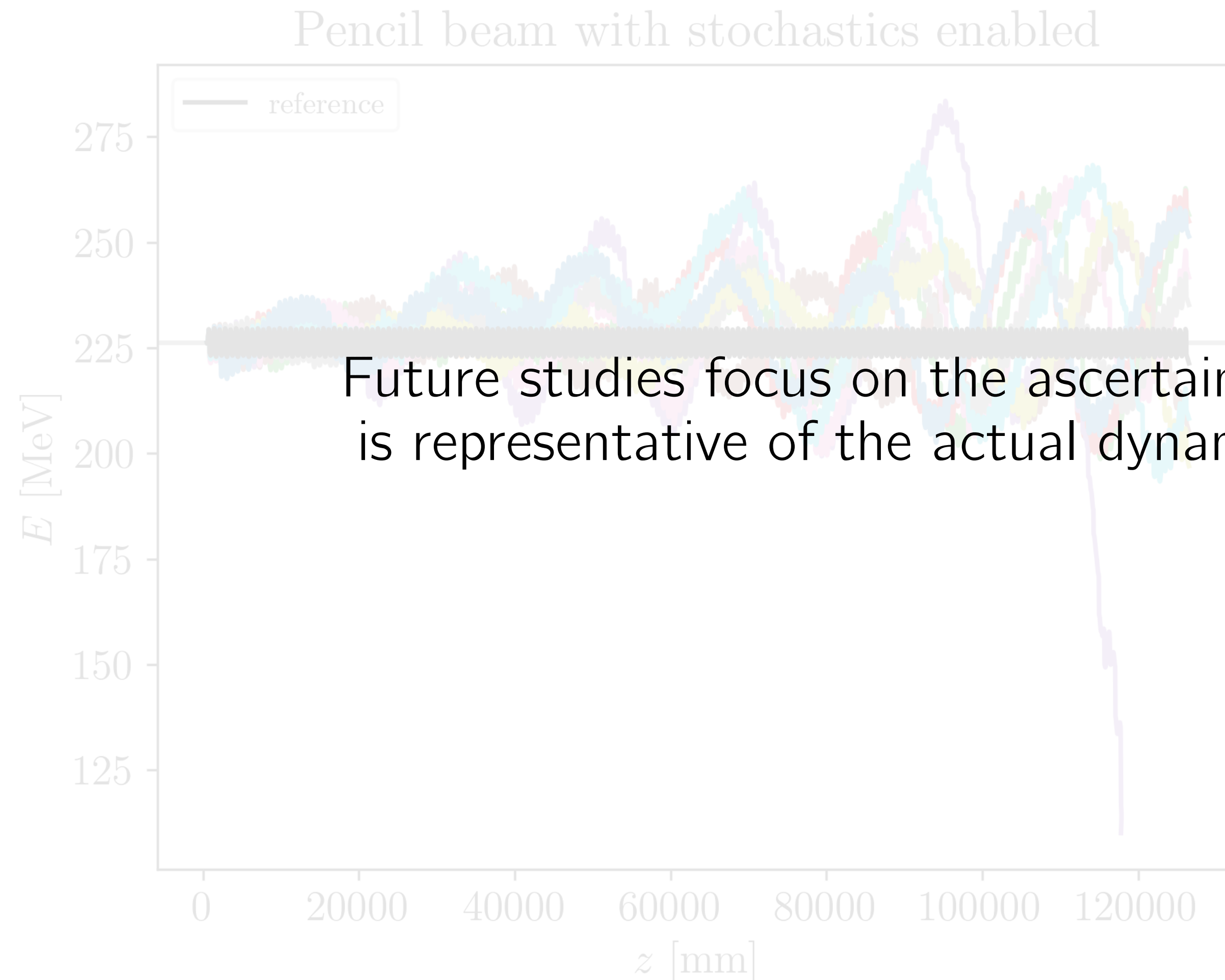
Pencil beam with stochastics enabled



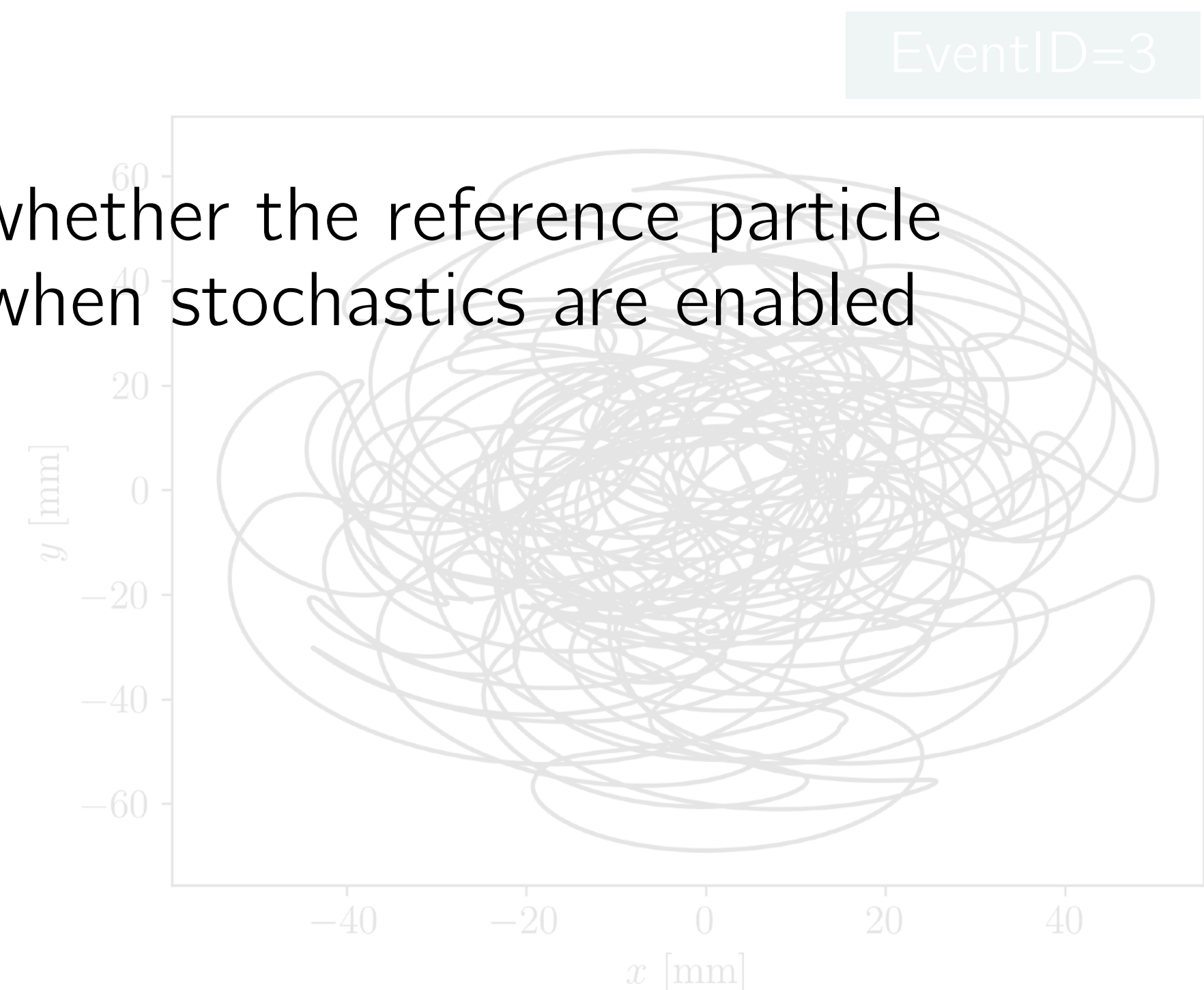
Enabling stochastics induces significant deviations from the reference trajectory



Toward a periodic design – next steps



Enabling stochastics induces significant deviations from the reference trajectory



Some other things I learned

How to calculate tune in a lattice w/
transverse and longitudinal coupling

Betatron tune is important to acceptance and transverse dynamics
==> Need to study in order to find appropriate lattice design when longitudinal dimension is introduced

HFOFO has coupling in 6D

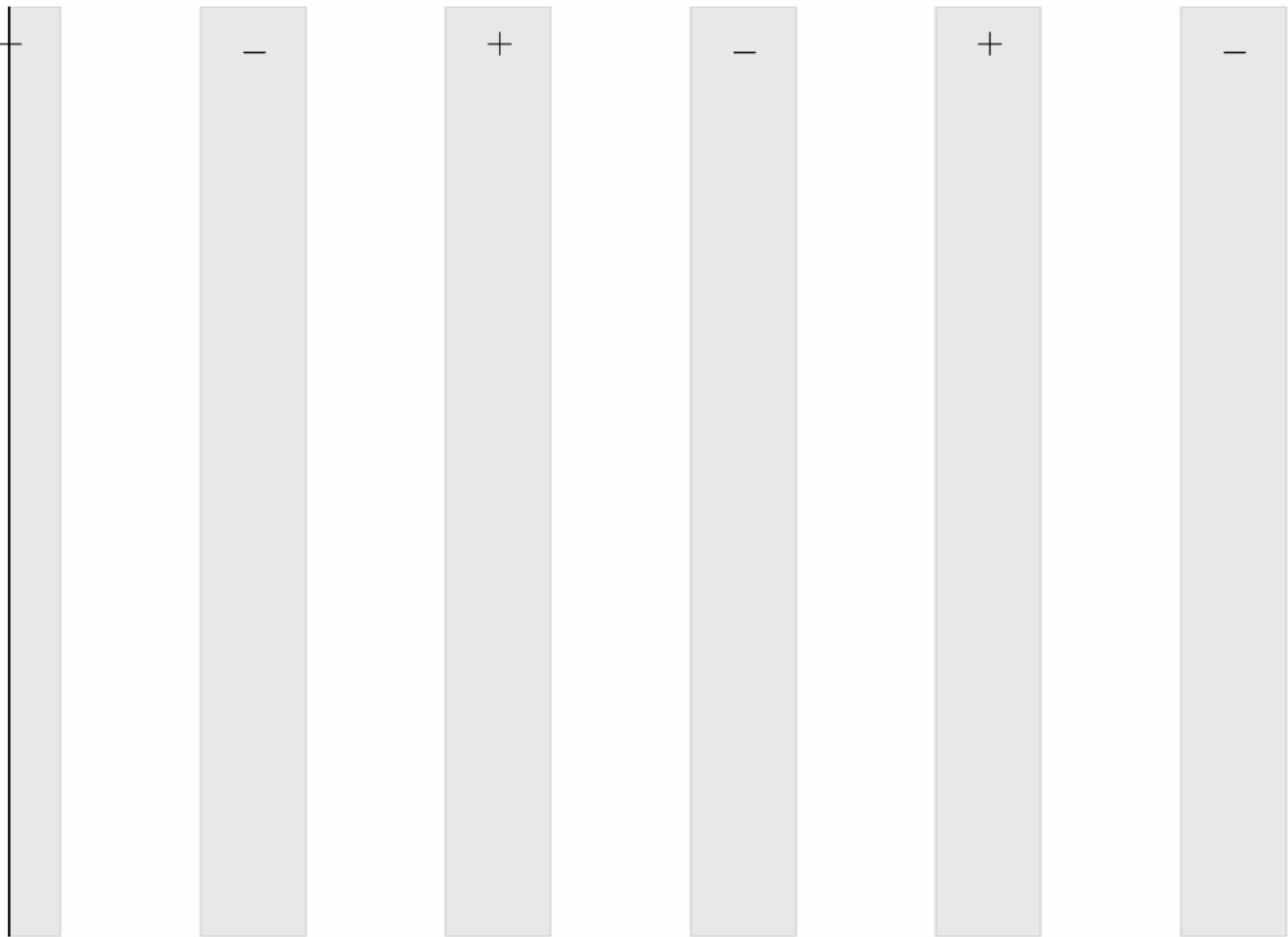
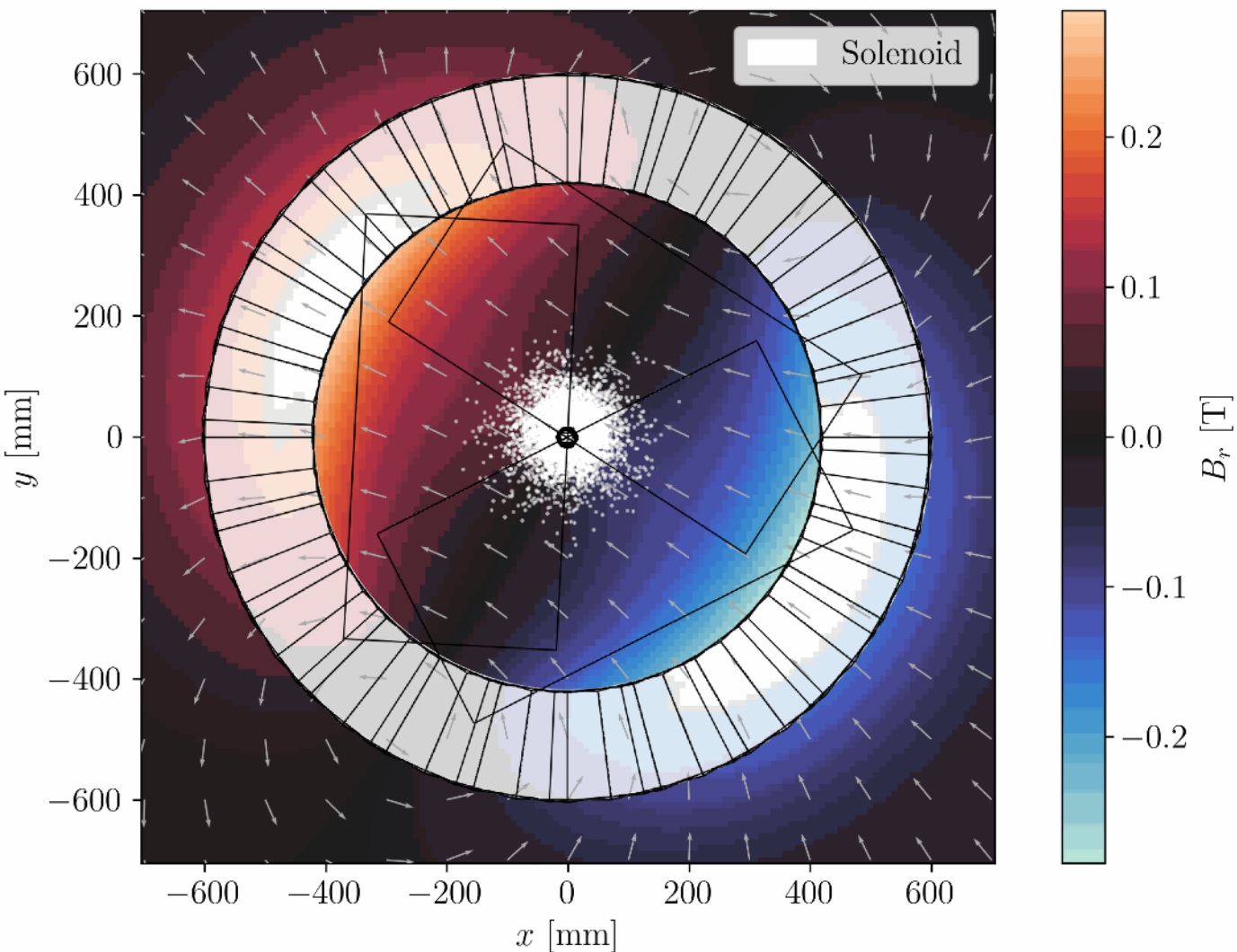
- ==> (1) Procedure for calculating tune is more involved;
(2) Also need to find synchro-betatron tune!

Open question:
can we replicate
Yuri's values?
—>

Table 1. Normal mode tunes and normalized equilibrium emittances.

Parameter	Mode I	Mode II	Mode III
Tune	$1.2271 + 0.0100i$	$1.2375 + 0.0036i$	$0.1886 + 0.0049i$
Emittance (mm)	2.28	6.13	1.93

The relationship between transverse beam dynamics and absorber orientations



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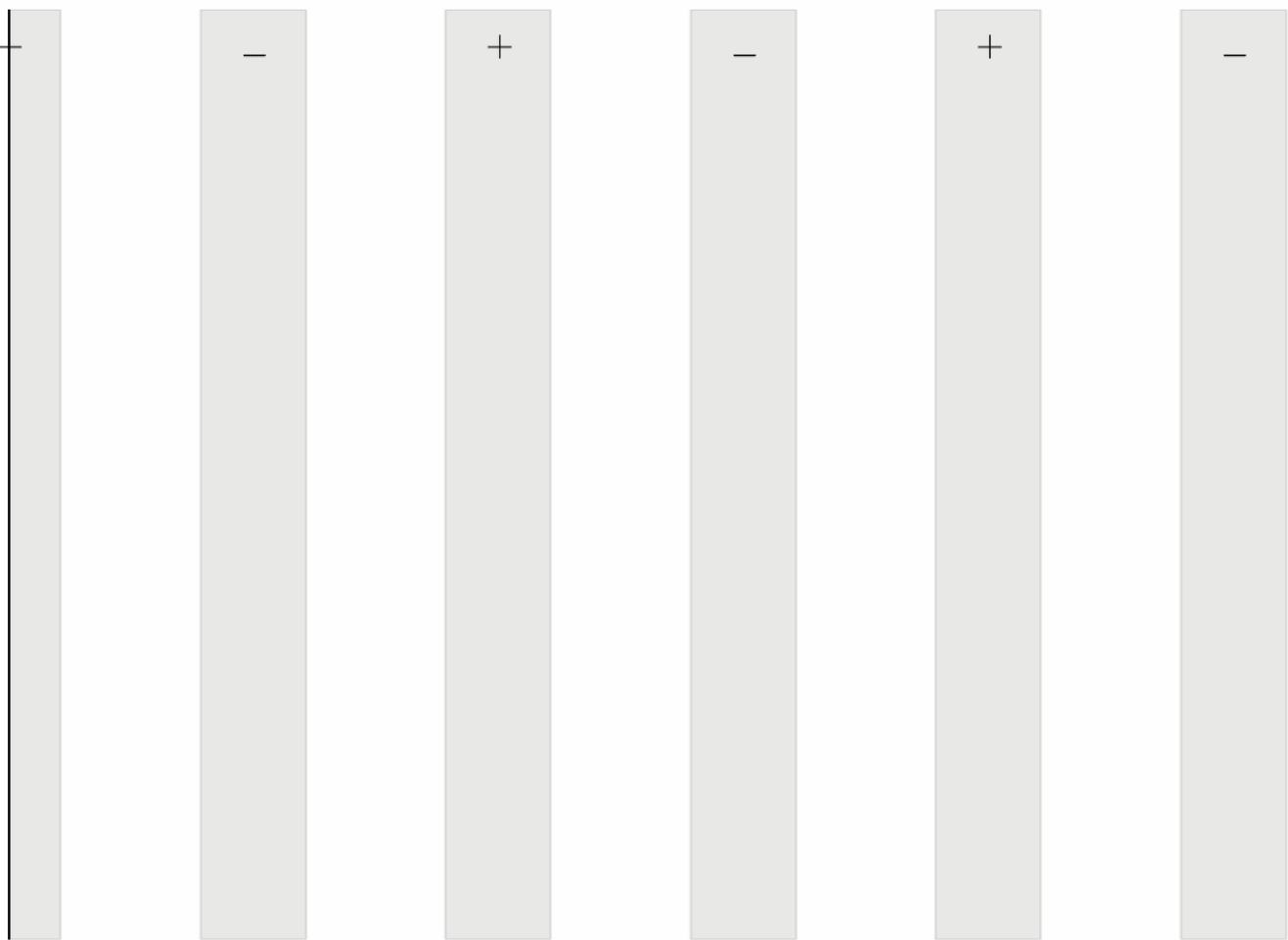
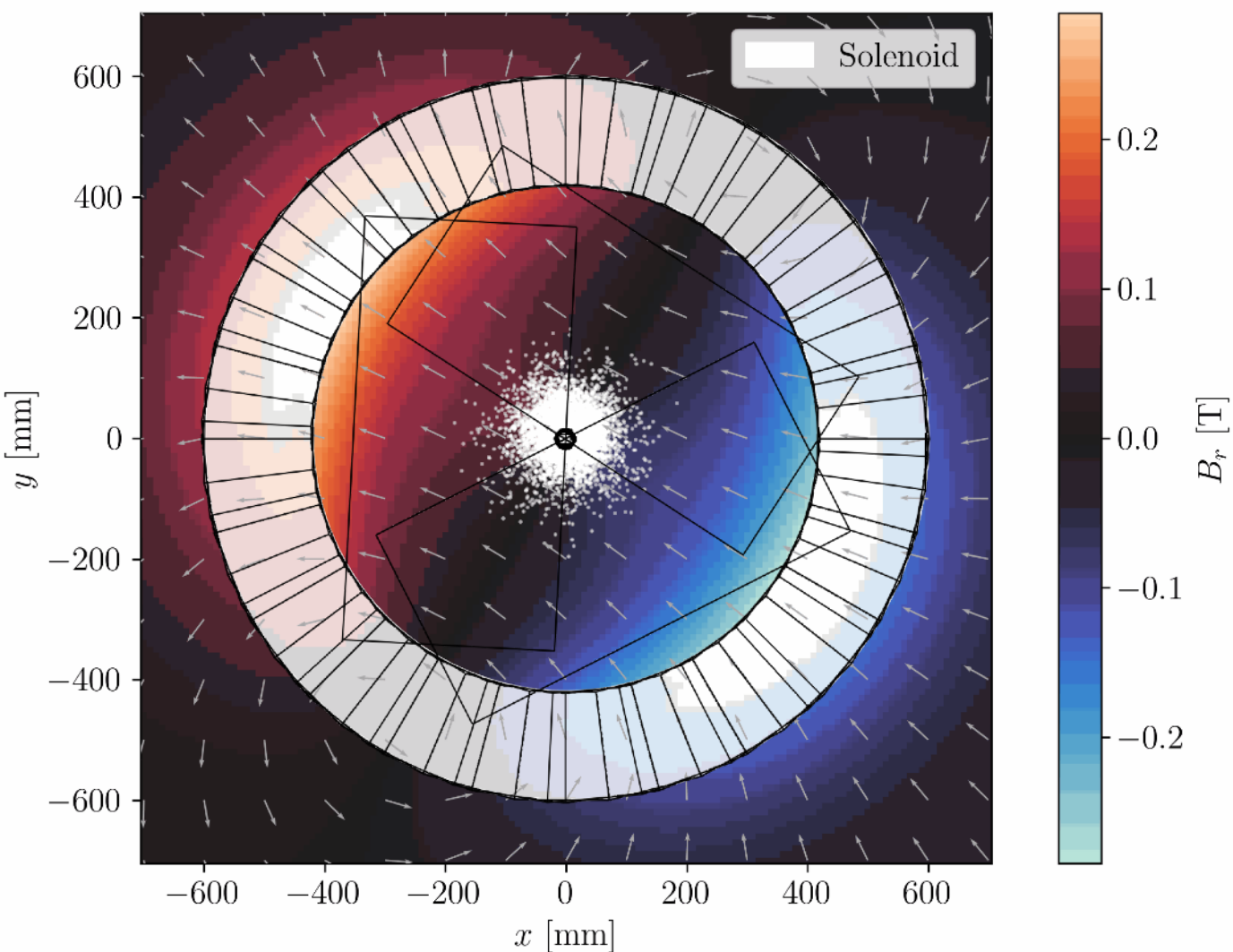
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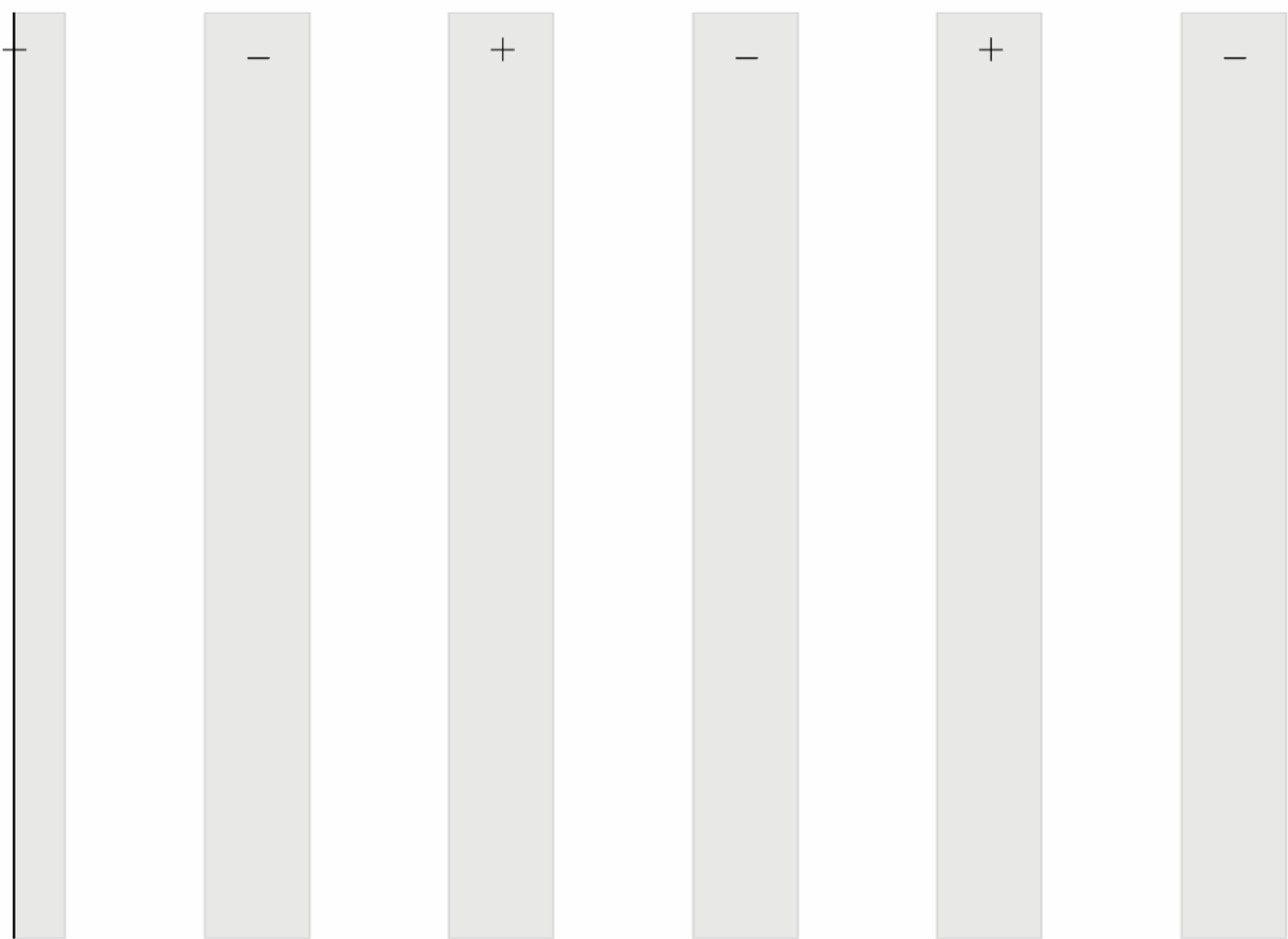
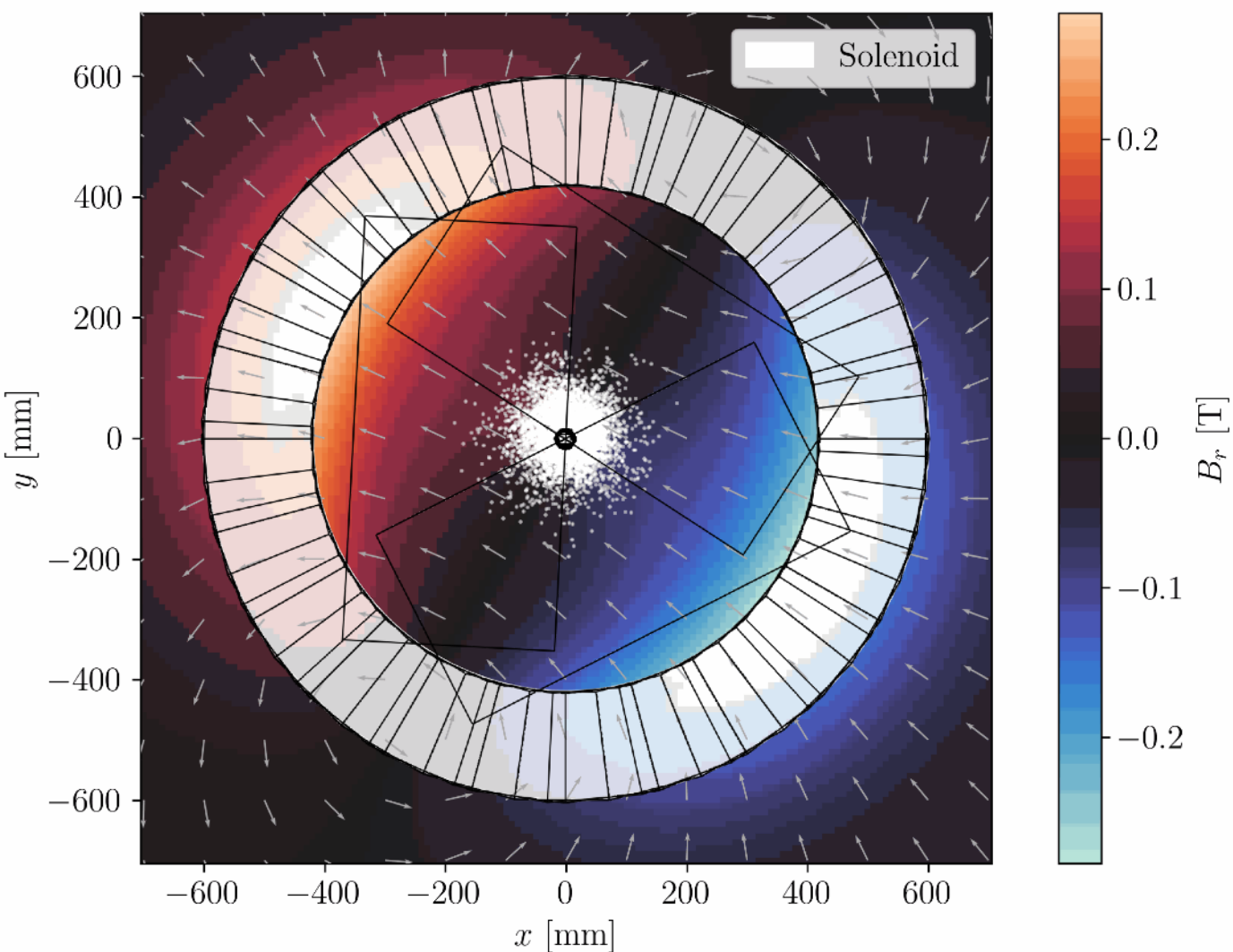
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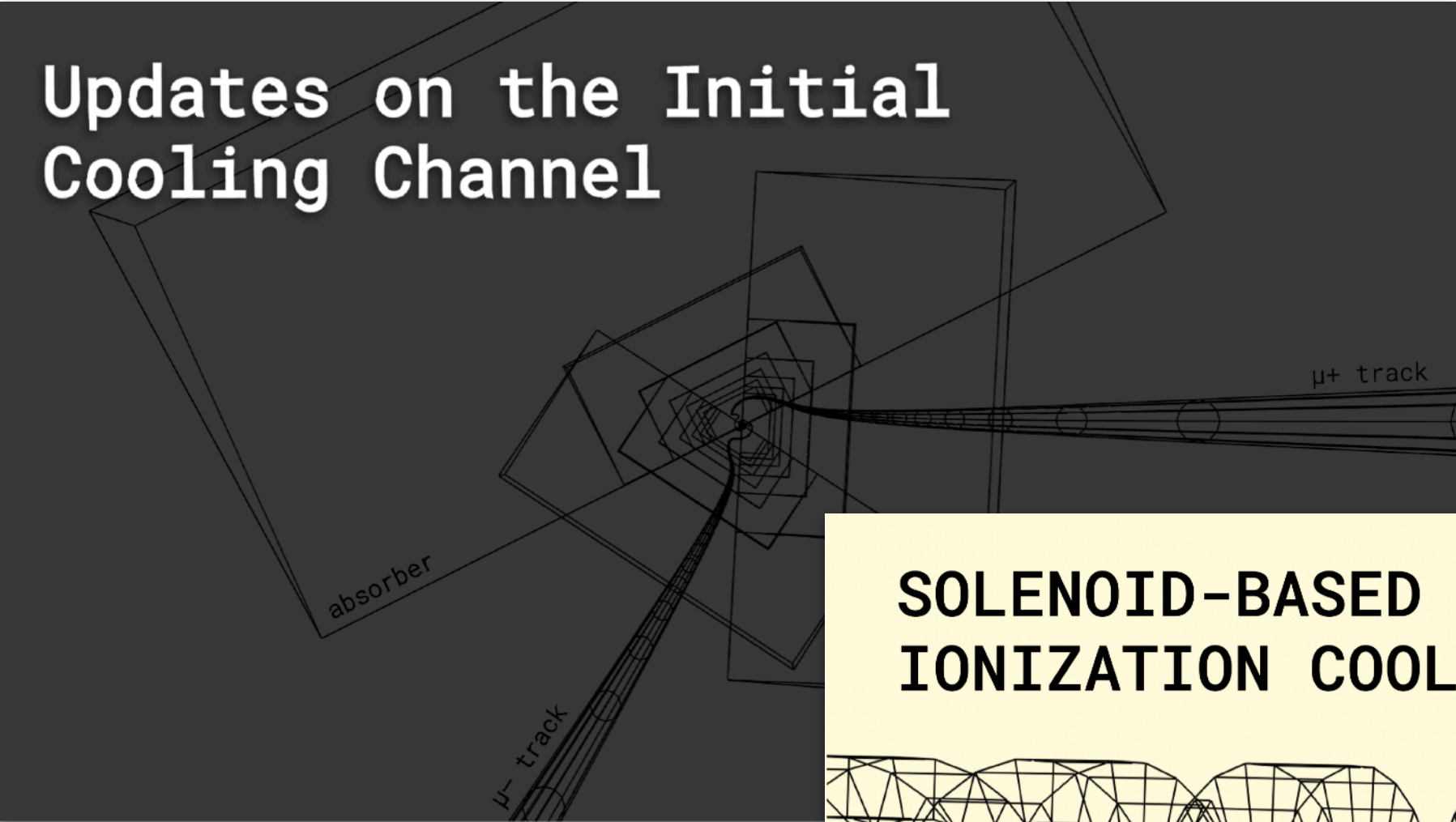
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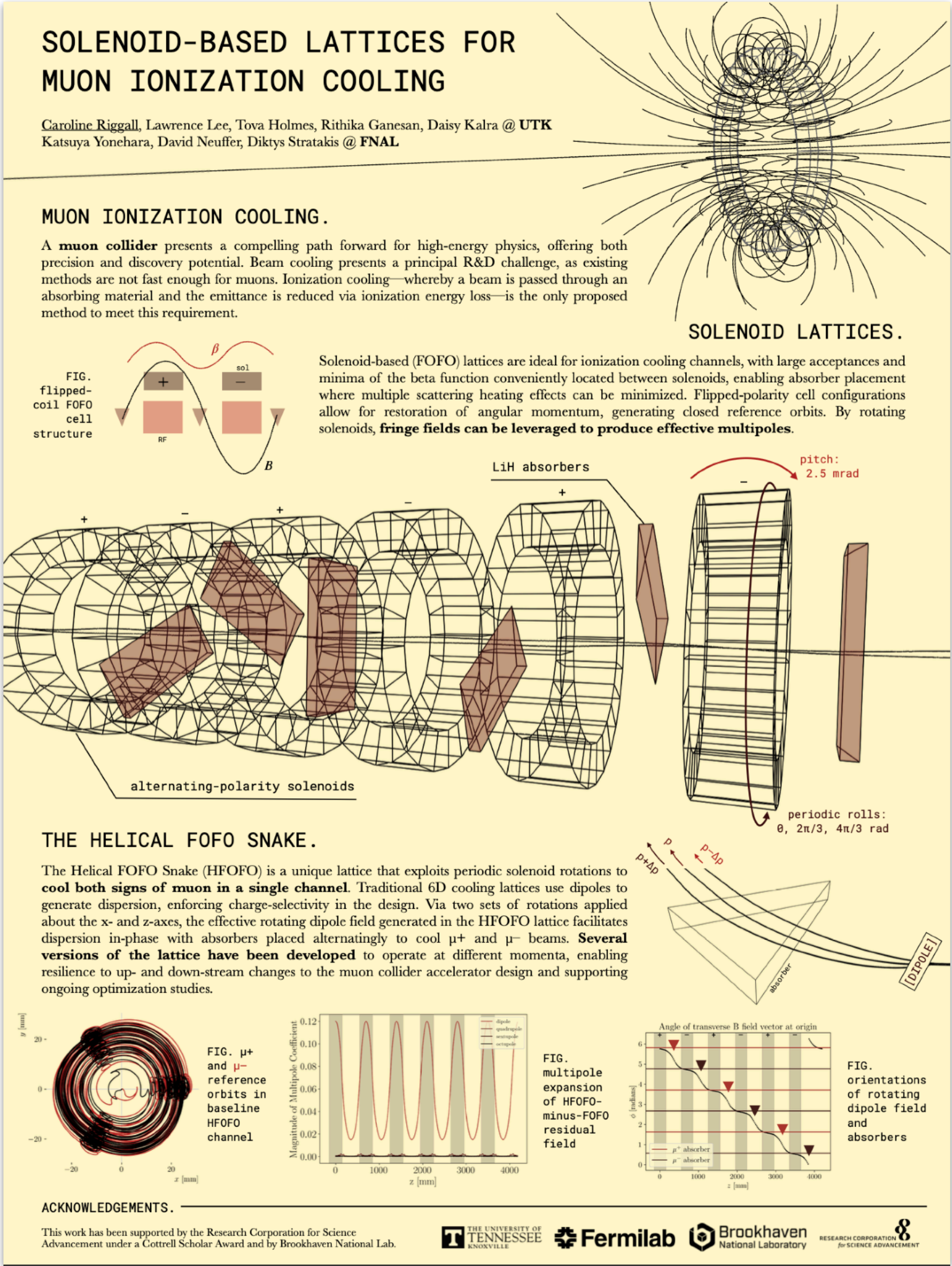
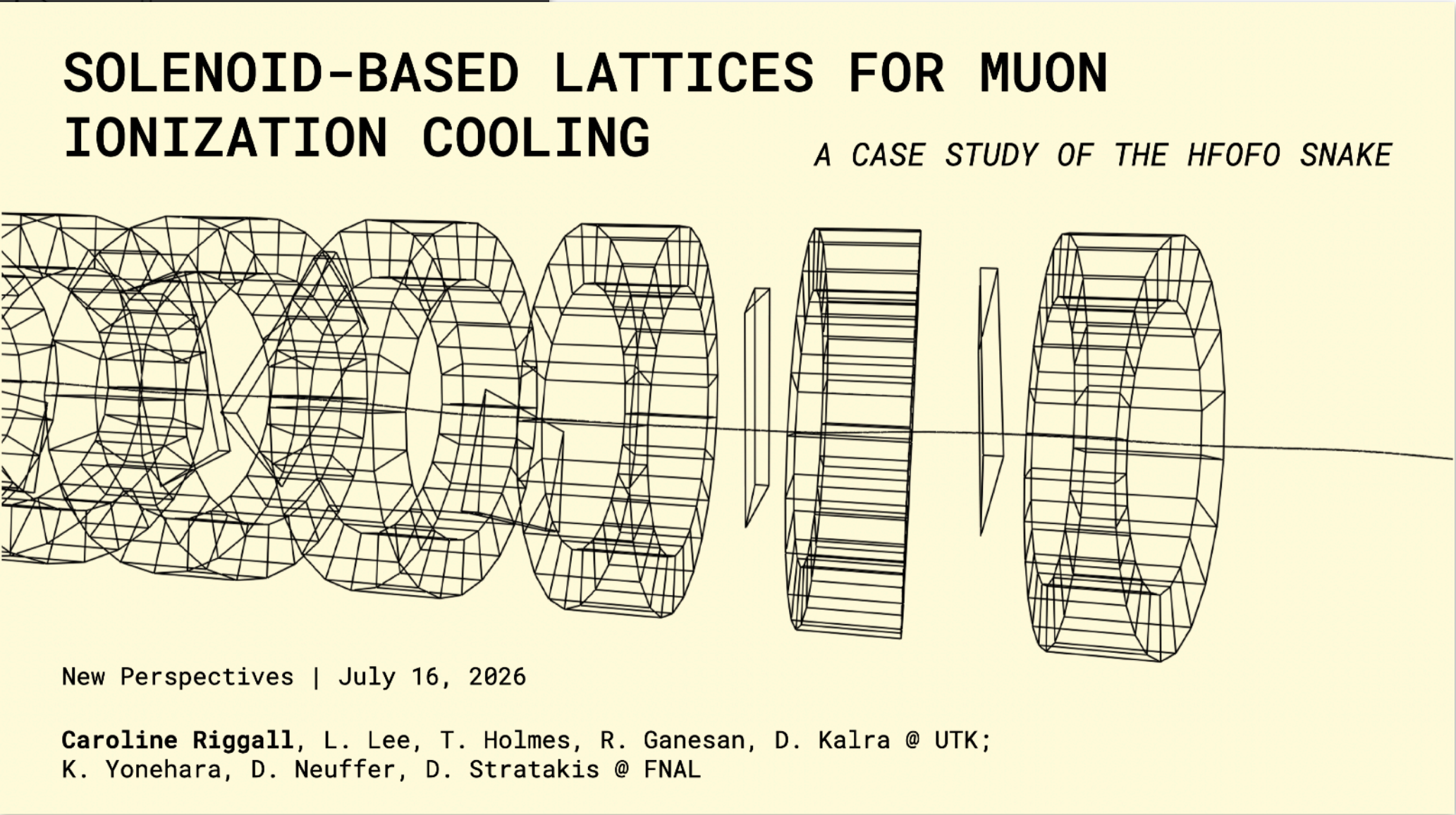


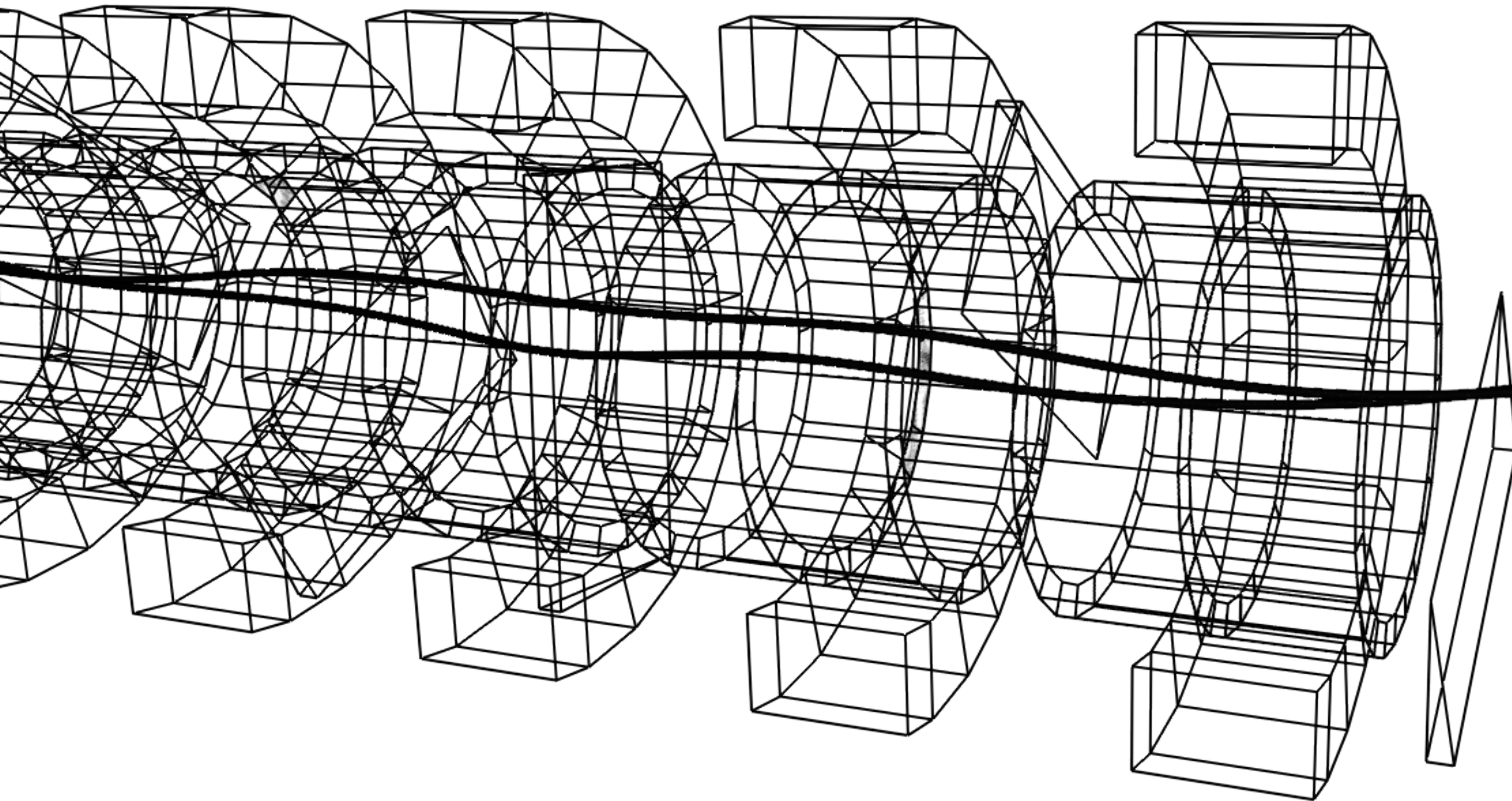
Some other things I did

Presented a poster
@ DPF Annual Meeting



Gave talks @ IMCC Meeting & FNAL New Perspectives





Thank μ !