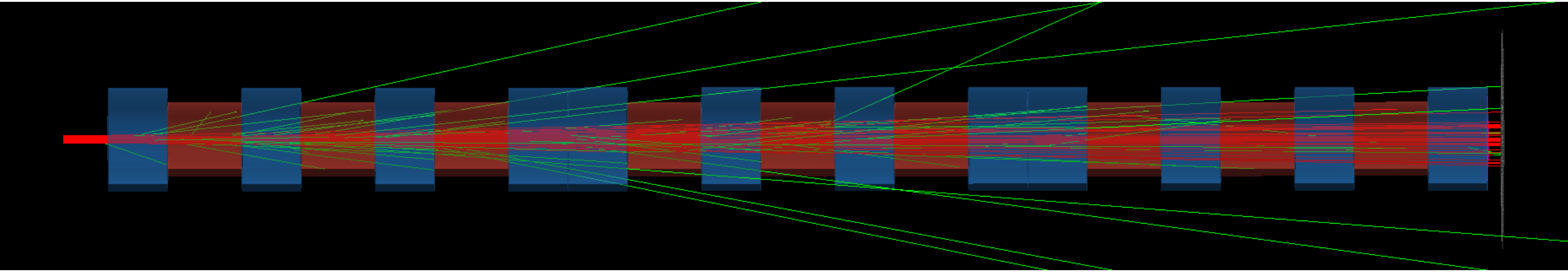
Studies on Muon Collider's Target with G4Beamline (11/24/2025)

Cheng-Hsu (Bryan) Nee

Pion Decay Channel (Triplet)



End

Twiss Parameters Calculation from Beam Optics

Goal. Compute (βx, αx) and (βy, αy) along the Triplet Pion Decay Channel directly from the **beam-optics model** (drifts + thick-lens quadrupoles). No dipoles here ⇒ D ≈ 0.

1) Element transfer matrices (thick lens)

For path length **L** and normalized quad strength **K = k1** (units m^{-2}):

Drift(L):

$$M_D = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$$

Quadrupole(L, K):

Focusing plane (sign of **K**):

if **K>0** ⇒

$$M_Q = \begin{pmatrix} \cos(\sqrt{K}L) & \dfrac{\sin(\sqrt{K}L)}{\sqrt{K}} \\ -\sqrt{K}\sin(\sqrt{K}L) & \cos(\sqrt{K}L) \end{pmatrix}$$

Defocusing plane (replace **K→−K**):

$$M_Q = \begin{pmatrix} \cosh(\sqrt{|K|}\,L) & \dfrac{\sinh(\sqrt{|K|}\,L)}{\sqrt{|K|}} \\ \sqrt{|K|}\,\sinh(\sqrt{|K|}\,L) & \cosh(\sqrt{|K|}\,L) \end{pmatrix}$$

2) Build the cell and the full lattice

A **triplet cell** = Drift(oL) → QF1(qL,KF) → Drift(oL) → QD(qL,KD) → Drift(oL) → QF2(qL,KF) → Drift(oL).

Full line = 3 identical cells (+ small end drift).

The **2×2 matrix** from the start to any s is the ordered product of element matrices up to s.

3) Propagate Twiss along z

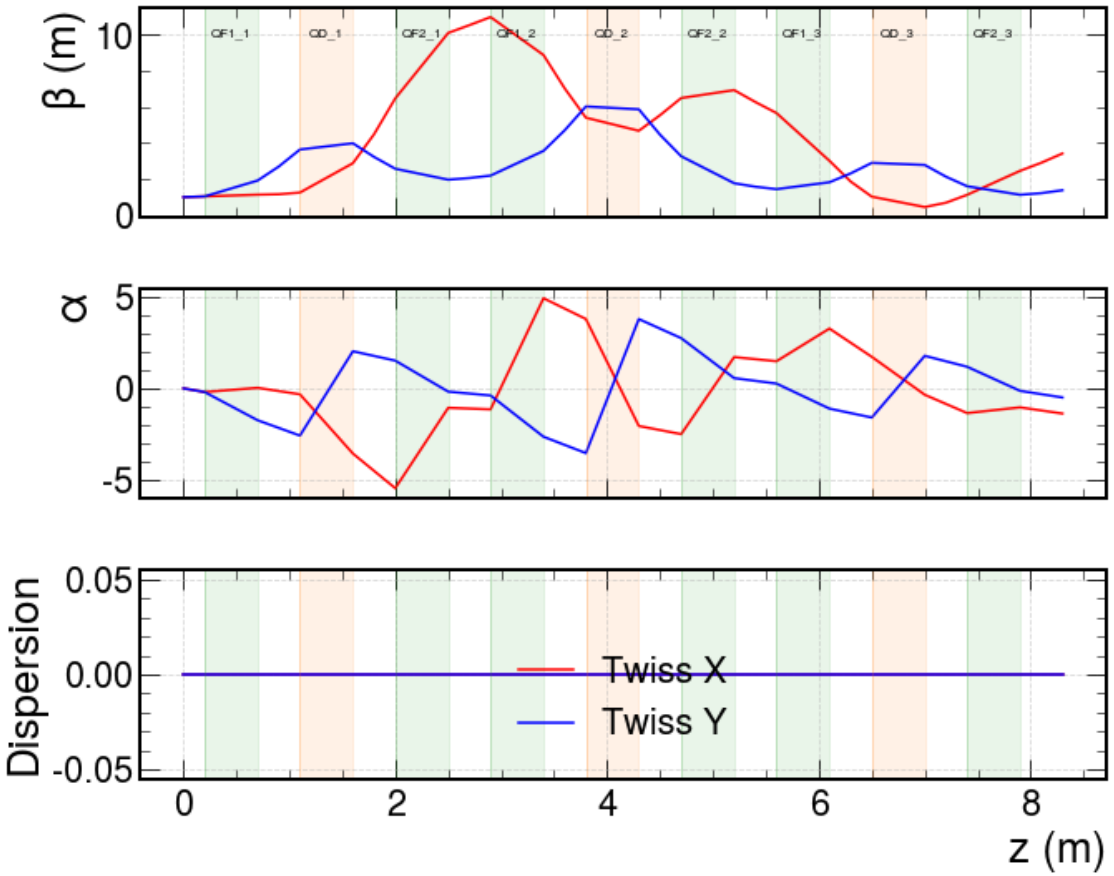
For each element with matrix **$M(s_k \rightarrow s_{k+1})$**:

Covariance propagation: $\Sigma_{k+1} = M\,\Sigma_k\,M^T$.

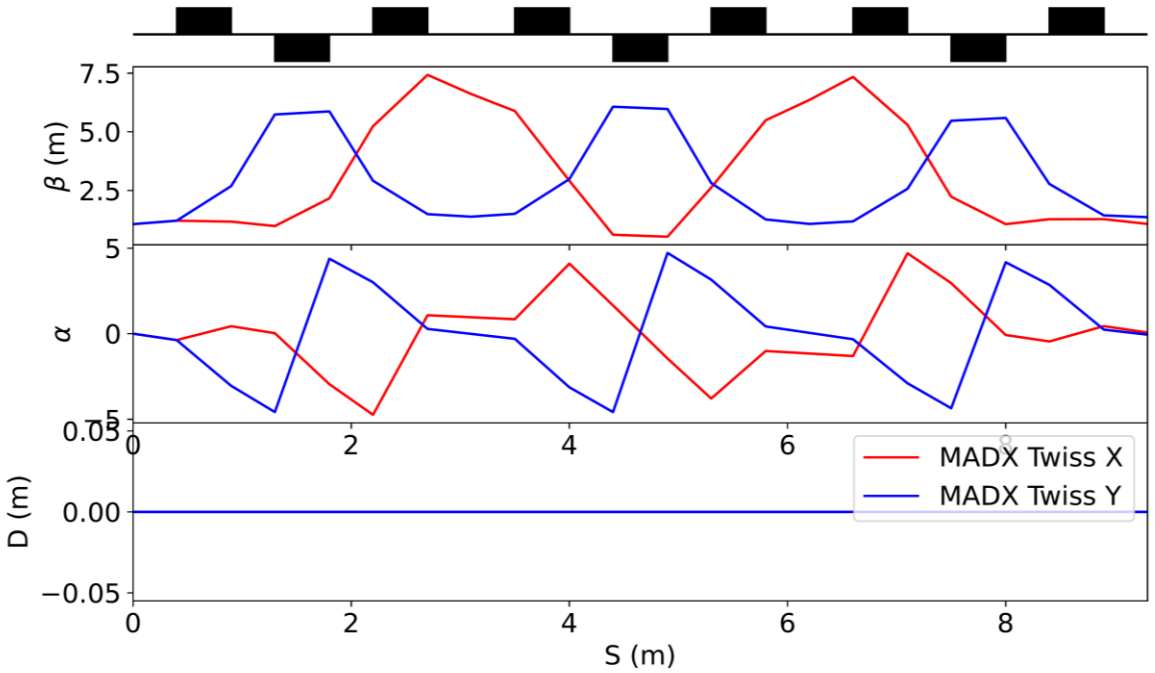
Recover Twiss: $\beta = \Sigma_{11},\ \alpha = -\Sigma_{12}$.

This gives the smooth β, α curves I plotted. I also sample at element edges/centers and shade QF/QD regions in the figure.

Twiss Parameters Calculation from Beam Optics



Mine



IMCC

Twiss Parameters Calculation from Second Moment

Define slopes (paraxial)

- Use *fixed* p_0(optics reference) to avoid momentum-coupling bias
- $x' = \frac{P_x}{p_0}, y' = \frac{P_y}{p_0}$

Momentum deviation & optional cut

- Relative deviation: $\delta = \frac{|\mathbf{p}| - p_0}{p_0}$
- Keep *core* beam only: $|\ \delta\ | \leq 0.25$

Remove horizontal dispersion (per slice)

- Regress linearly (least squares):
- $x \approx D\,\delta, x' \approx D'$
- Subtract fitted pieces: $x_\beta = x - D\delta,\ \ x'_\beta = x' - D'\delta$

Compute centered second moments (betatron)

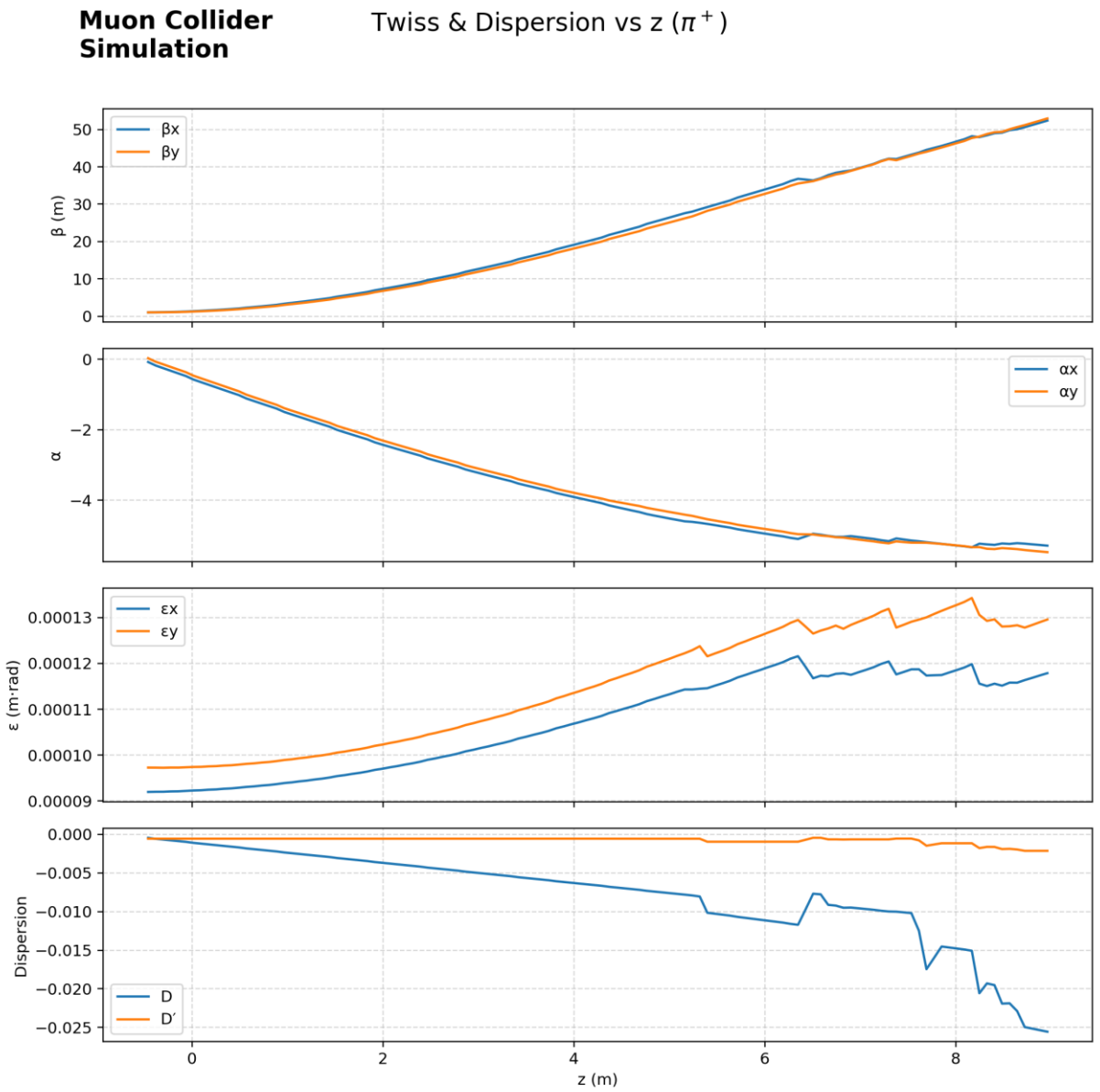
- With mean-centering, per slice: $\left\langle x_\beta^2 \right\rangle, \left\langle {x'_\beta}^2 \right\rangle, \left\langle x_\beta x'_\beta \right\rangle$

Twiss from moments (per slice)

- Determinant: $\Delta = \langle x_\beta^2 \rangle \langle {x'_\beta}^2 \rangle - \langle x_\beta x'_\beta \rangle^2$
- Geometric emittance: $\varepsilon_x = \sqrt{\Delta}$

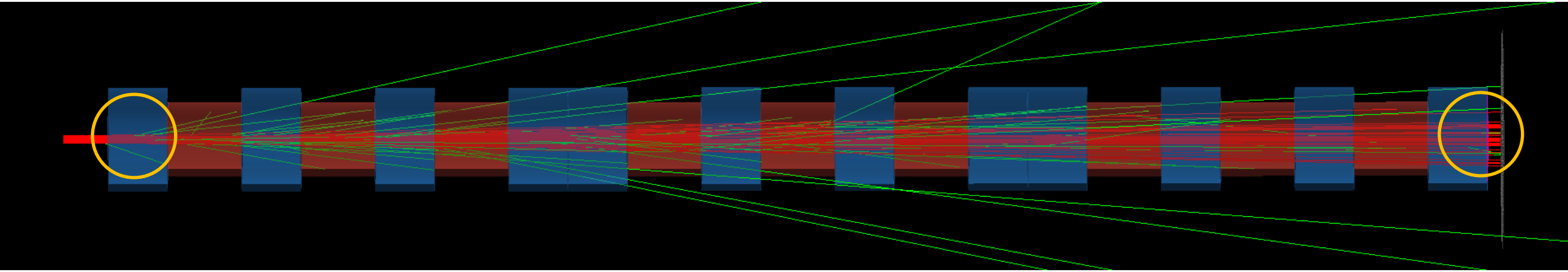
Beta/alpha:

- $\beta_x = \frac{\langle x_\beta^2 \rangle}{\varepsilon_x}, \alpha_x = -\frac{\langle x_\beta x'_\beta \rangle}{\varepsilon_x}$
- Repeat for $y \rightarrow (\varepsilon_y, \beta_y, \alpha_y)$



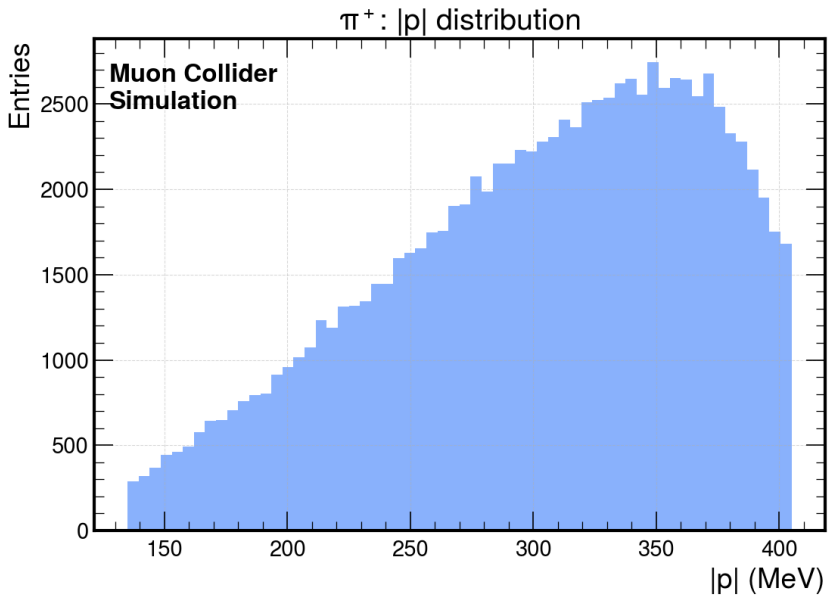
Pion Decay Channel (Triplet)

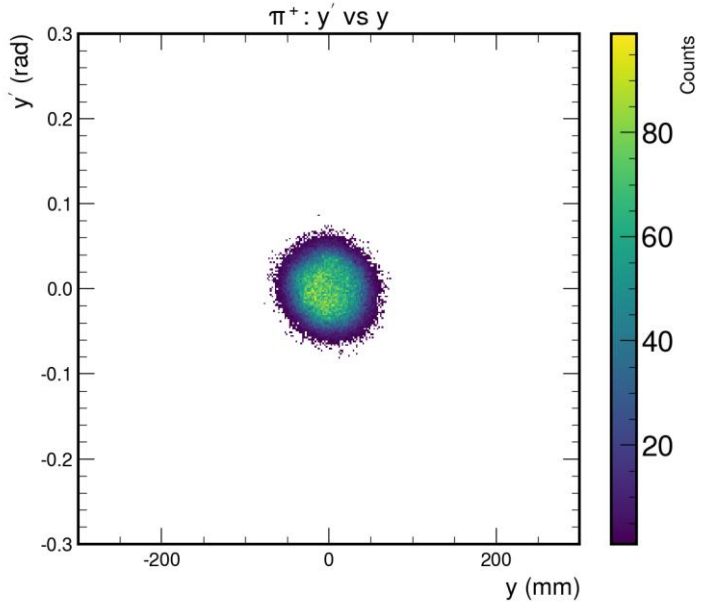
Front

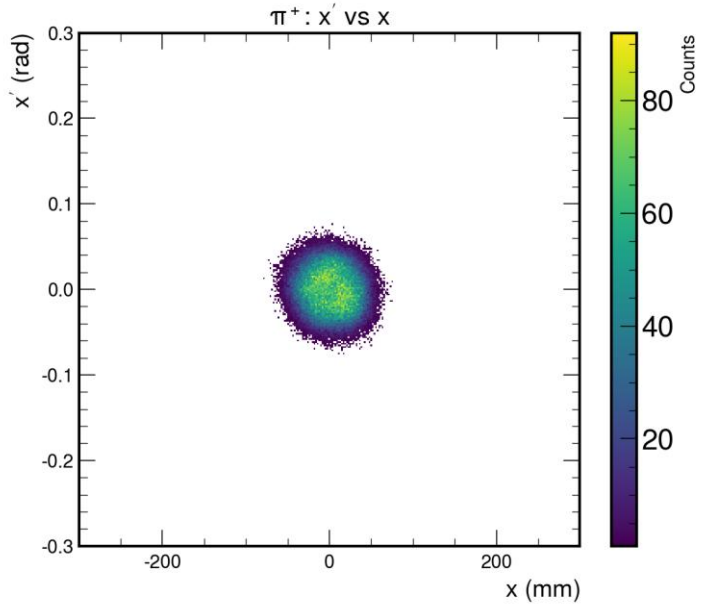


End

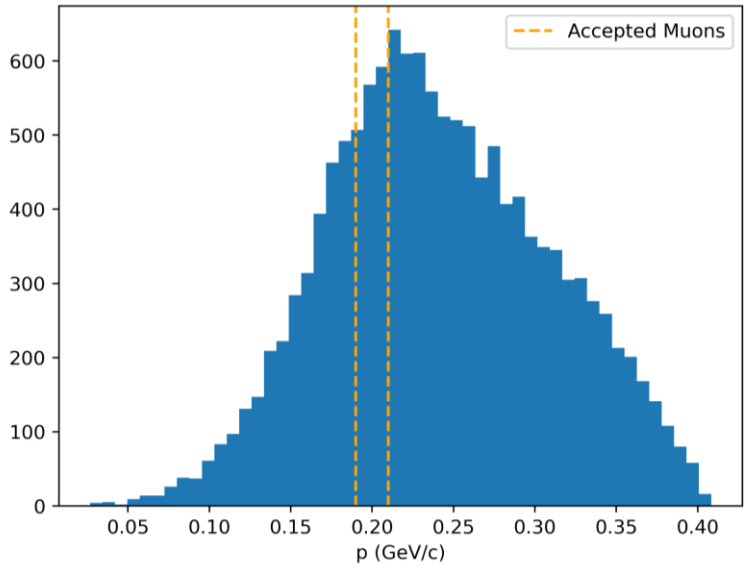
Convert BDSIM Input to G4Beamline Input

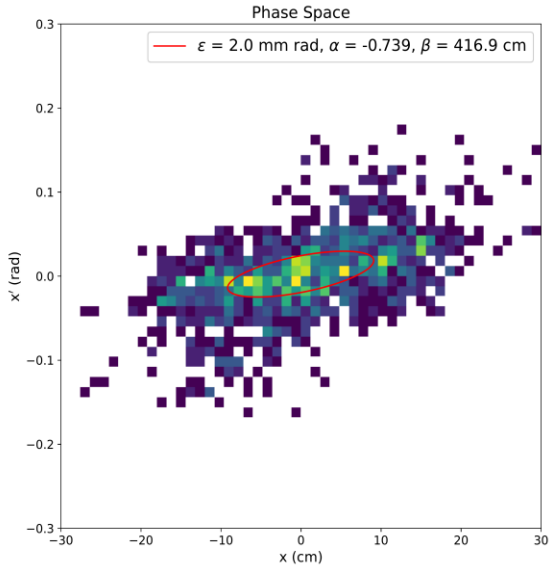


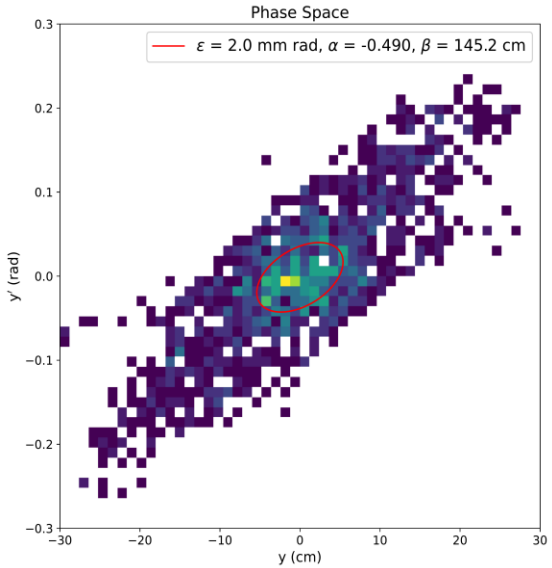


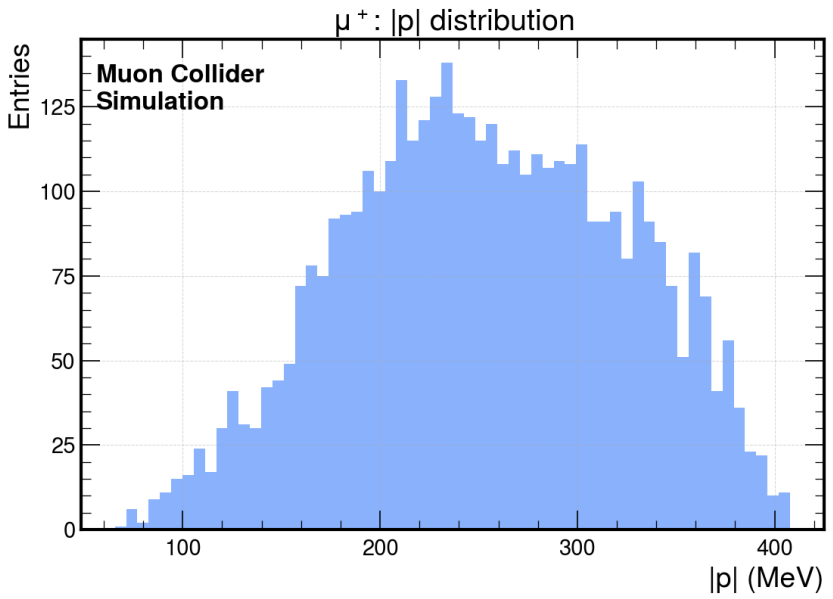


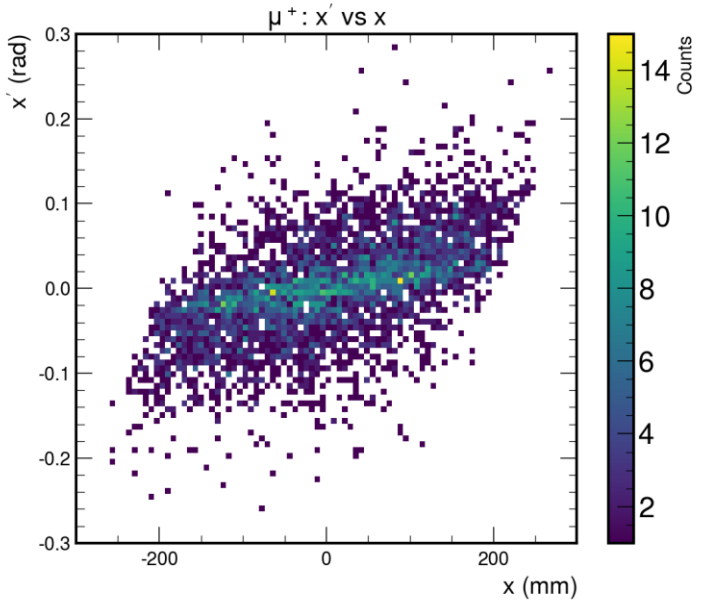
End of the Decay Channel (Top: IMCC, Bottom: Mine)

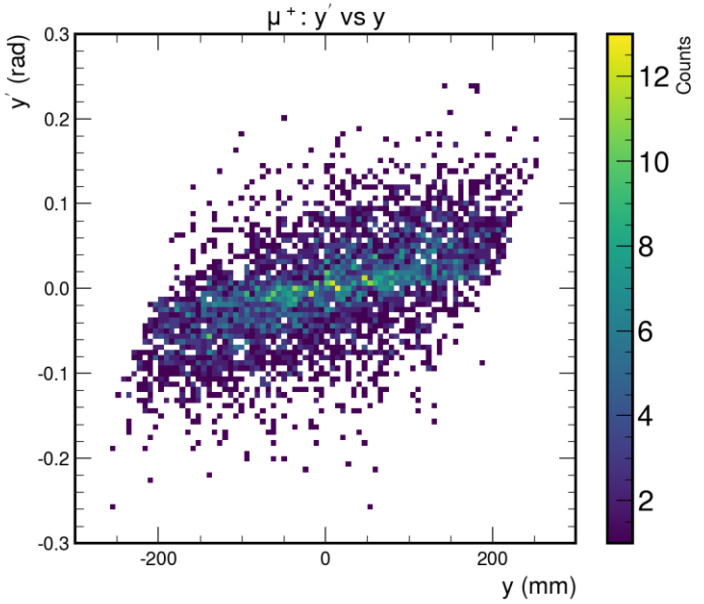


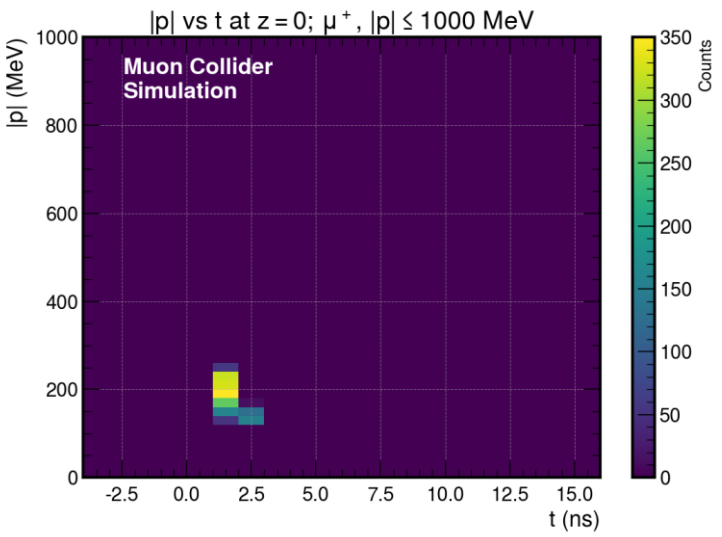


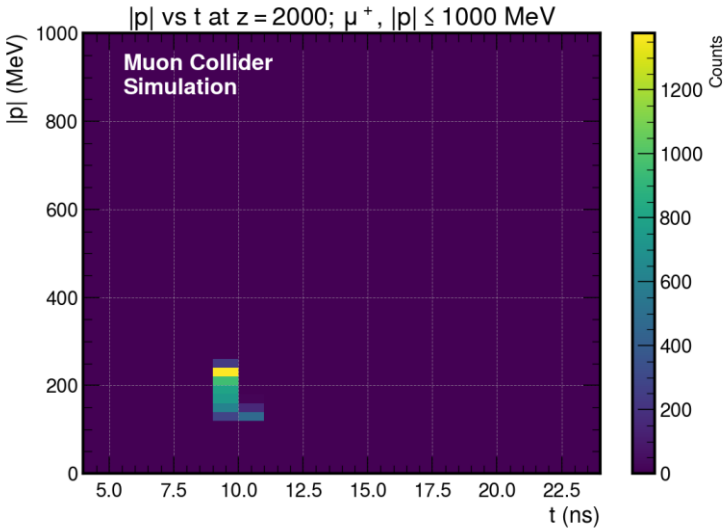


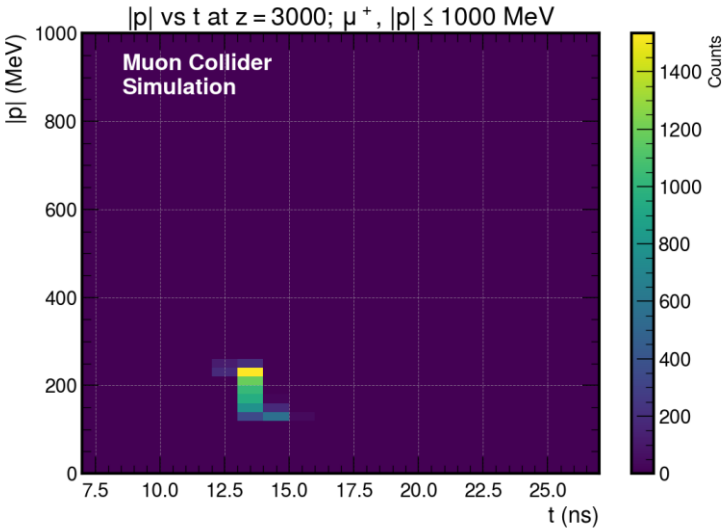


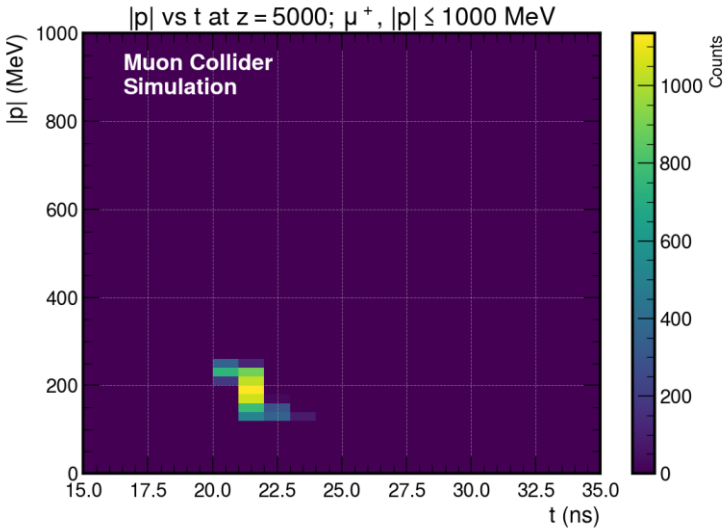


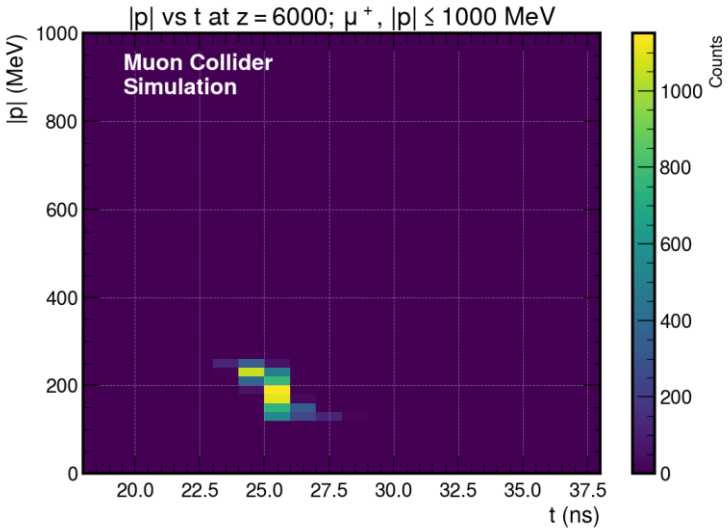



|p| vs t at z = 0; μ⁺, |p| ≤ 1000 MeV
Muon Collider Simulation
|p| (MeV)
t (ns)
Counts


|p| vs t at z = 2000; μ⁺, |p| ≤ 1000 MeV
Muon Collider Simulation
|p| (MeV)
t (ns)
Counts


|p| vs t at z = 3000; μ⁺, |p| ≤ 1000 MeV
Muon Collider Simulation
|p| (MeV)
t (ns)
Counts


|p| vs t at z = 5000; μ⁺, |p| ≤ 1000 MeV
Muon Collider Simulation
|p| (MeV)
t (ns)
Counts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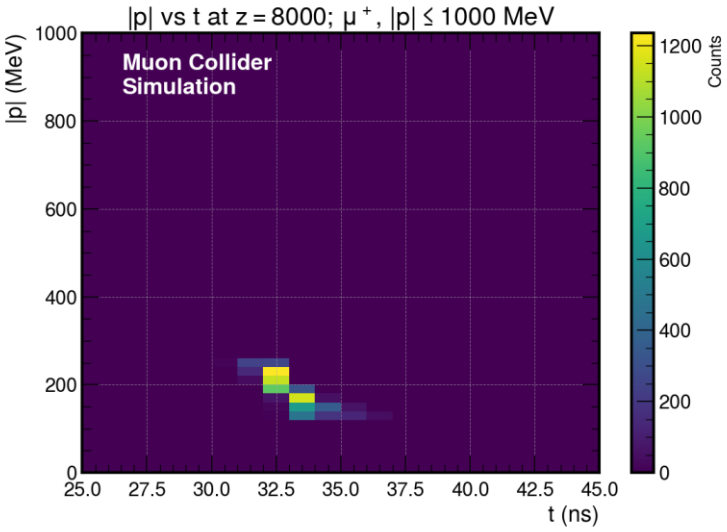
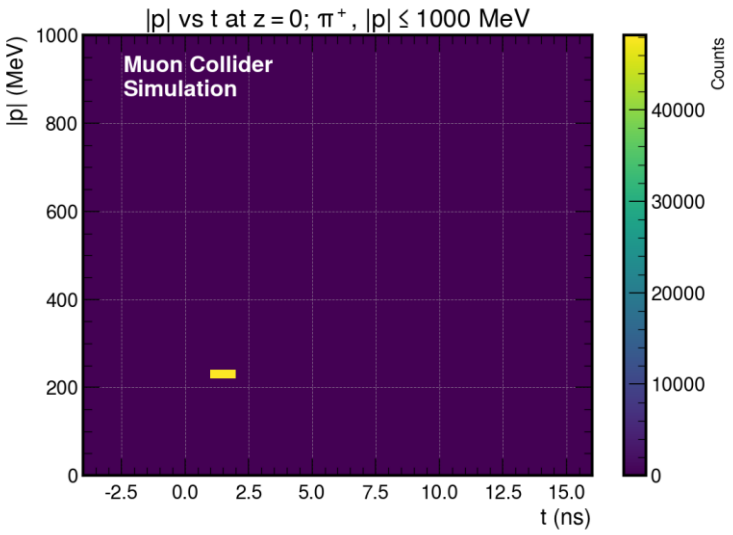
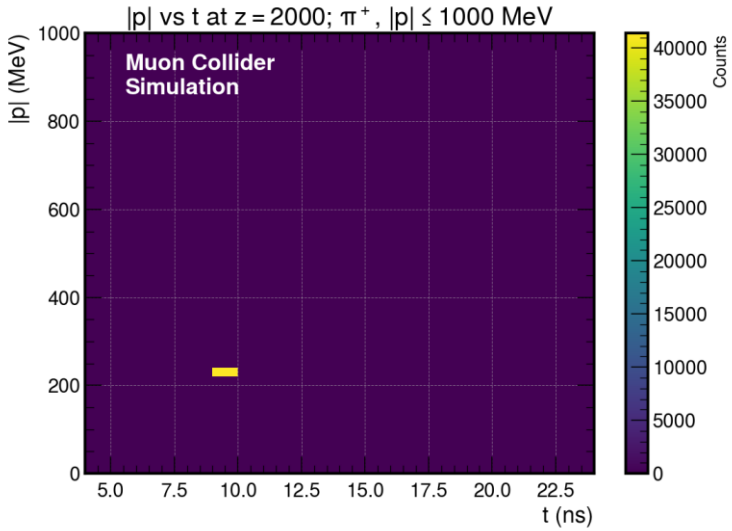
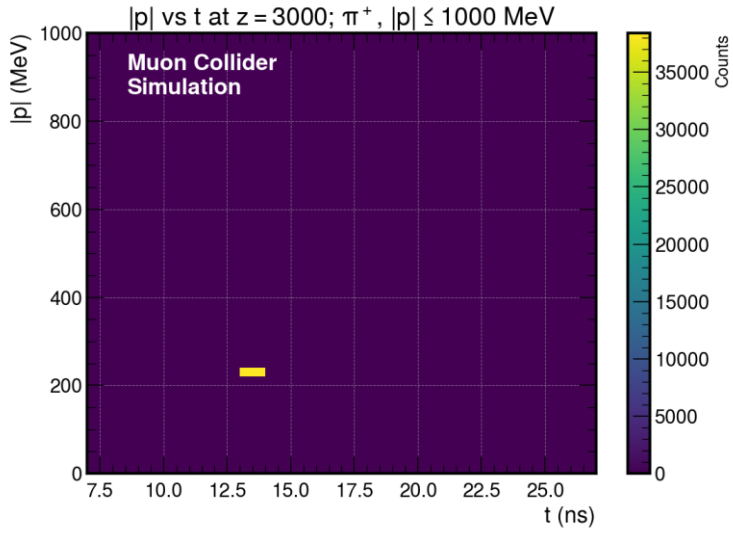
Muon Collider Simulation
|p| (MeV)
t (ns)
Counts


|p| vs t at z = 8000; μ⁺, |p| ≤ 1000 MeV
Muon Collider Simulation
|p| (MeV)
t (ns)
Counts


|p| vs t at z = 0; π⁺, |p| ≤ 1000 MeV
Muon Collider Simulation
|p| (MeV)
t (ns)
Counts


|p| vs t at z = 2000; π⁺, |p| ≤ 1000 MeV
Muon Collider Simulation
|p| (MeV)
t (ns)
Counts


|p| vs t at z = 3000; π⁺, |p| ≤ 1000 MeV
Muon Collider Simulation
|p| (MeV)
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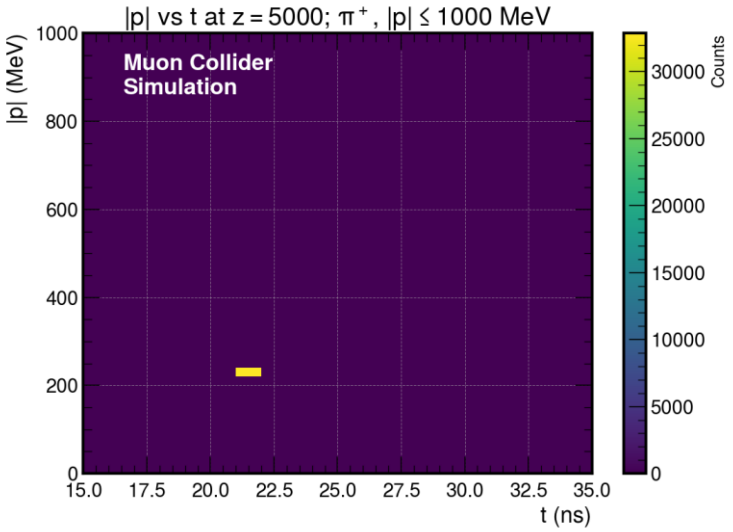
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Muon Collider Simulation
|p| (MeV)
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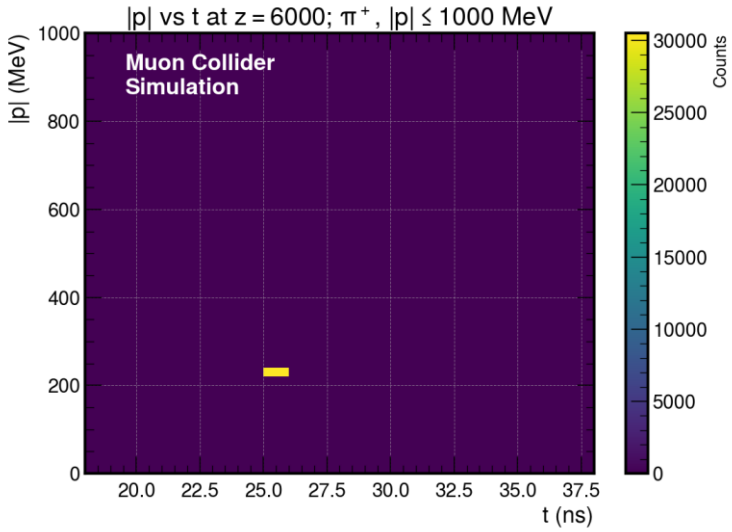
Counts


|p| vs t at z = 6000; π⁺, |p| ≤ 1000 MeV
Muon Collider Simulation
|p| (MeV)
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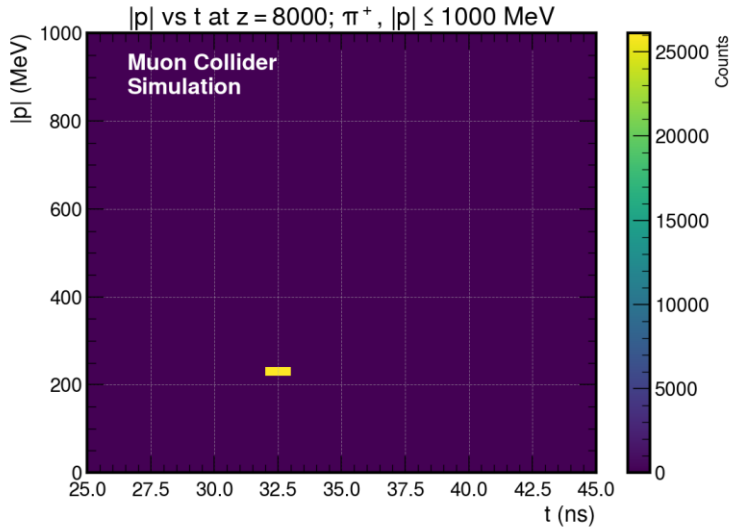
|p| vs t at z = 8000; π⁺, |p| ≤ 1000 MeV
Muon Collider Simulation
|p| (MeV)
t (ns)
Counts